Math 200a Fall 2011 Homework 3

Due Friday 10/14/2011 by 5pm in homework box on 6th floor of AP&M

Reading assignment: Sections 4.4-4.6, 5.1.

Exercises:

All exercise numbers refer to Dummit and Foote, 3rd edition. I like to list extra exercises from the text, but *only the exercises marked with a star are to be handed in for grading.* I include these extra exercises because they seem interesting or useful but I cannot include them without having too many assigned exercises. So please at least look over the unstarred exercises. I also frequently assign some exercises not from the text; usually these are all starred and thus to be handed in.

Section 4.3 #5*, 8, 11, 13, 19*, 20*, 21*, 23, 24, 25, 26, 30, 33*, 34

Comments:

#19 is more or less immediate from from 4.1 #9. Since you did that exercise on the last homework, you may do #19 directly by quoting 4.1 #9.

For #21, it is very important that you include cycles of length 1 in the cycle type. So (1)(2)(345) has cycle type $1, 1, 3$ and so according to this exericse, will commute with an odd permutation since the integers in the cycle type are not distinct.

Section 4.4 #1, 11, 18*, 19

Section 4.6 #1* (hint: 4.2 #8), 2*, 3

Exercises not from the text: (all to be handed in):

1*. Consider the group G of all rotations of a cube in 3-space (i.e. rotations so that the cube occupies the same region of space afterwards), with operation being composition of rotations. First show that $|G| = 24$, and then prove that $G \cong S_4$.

Hint: To prove $|G| = 24$, consider two vertices joined by an edge and think about the different places those vertices can go to. For the second statement, consider the four pairs of diagonally opposite vertices, i.e. pairs such that the line joining those vertices goes through the center of the cube. Consider what rotations do to those pairs.

2*. Let Q be the quaternion group of order 8. Show that $\mathrm{Aut}(Q) \cong S_4$.

Hint: Q has 6 elements of order 4, $\pm i \pm j, \pm k$. Show that any automorphism of Q is determined by how those 6 elements are permuted. Determine which permutations actually give an automorphism using the presentation for Q that you proved in Exercise 6.3 #7 on homework 2. Then relate the possible permutations to the way the 6 *faces* of a cube are permuted by the rotation group of a cube. Alternatively, you could find a presentation for S_4 and use this to show the group you have found is isomorphic to S_4. Yet other methods are possible.