Math 200a Fall 2011 Homework 4

Due Friday 10/28/2011 by 5pm in homework box on 6th floor of AP&M

Upcoming schedule: We will have a midterm exam on Wed. Nov. 2 in class. There will be no homework due on Friday of that week (Nov. 4). There will be a homework due the next week (Nov. 11) even though we have no class that day.

Reading assignment: Beginning of 5.4, 6.1. We will not cover 6.2.

Exercises to be handed in: (all exercise numbers refer to Dummit and Foote, 3rd edition.)

Section 5.5: **2**, **12**

Remark: Do additional exercise 1 below before doing exercise 5.5 #12 above. In 5.5 #12, I do want you to show that there are exactly 5 distinct groups up to isomorphism. Use additional exercise 1 to help you prove that some of the semi-direct products you get in 5.5 #12 are isomorphic to each other; this will allow you to show there are at most 5 distinct groups of order 20. You still need to check that the 5 you found are non-isomorphic.

Section 6.1: **4**, **9**, **10**.

Exercises not from the text: (all to be handed in):

1. Let K and H be groups, and let $\phi_1 : K \to \mathrm{Aut}(H)$ be a homorphism. Let $G_1 = H \rtimes_{\phi_1} K$ be the semi-direct product.

Suppose that $\sigma : K \to K$ is an automorphism of K. Then $\phi_2 = \phi_1 \circ \sigma : K \to \mathrm{Aut}(H)$ is also a homomorphism, and so we can also consider the semidirect product $G_2 = H \rtimes_{\phi_2} K$.

(a). Prove that the groups G_1 and G_2 are isomorphic.

(b). Use part (a) to justify the claim made in class that if with $p < q$ are primes with $p|(q-1)$, then there are precisely two groups of order pq up to isomorphism.

2. A group G is called *polycyclic* if G has a series

$$G_0 = \{e\} \trianglelefteq G_1 \trianglelefteq \cdots \trianglelefteq G_{n-1} \trianglelefteq G_n = G$$

such that for all $1 \leq i \leq n$, G_i/G_{i-1} is a cyclic group (finite or infinite).

(a). Show that if G is polycyclic, then any subgroup or factor group of G is again polycyclic.

(b). Show that if $N \trianglelefteq G$ and G/N and N are polycyclic groups, then G is also polycyclic.