

Math 200a Fall 2012 Homework 5

Due Friday 11/9/2012 by 12pm in homework box

Upcoming schedule: We will have a review on Monday 10/29 (send Johanna Hennig your suggestions.) Wednesday 10/31 is the exam, which covers up through semi-direct products. For the exam, you should understand the definition of semidirect product and the recognition theorem for semidirect products, and have a basic understanding of how to analyze the possible ψ occurring in a semidirect product $H \rtimes_{\psi} K$. The more subtle question of classifying the possible $H \rtimes_{\psi} K$ up to isomorphism is addressed in the homework problems on this problem set, but will not be on the exam. Friday 11/2 will be a guest lecture by Professor Salehi-Golsefidy.

Note that this homework is not due until the end of the following week (i.e., you get a break.) We will cover the material on nilpotent groups which is relevant for some of the exercises below on Monday 11/5, but of course you can also read ahead.

Reading assignment: Beginning of 5.4 on commutators, 5.5, 6.1. You may find 6.2 interesting reading, but the last material on groups we will cover is in 6.1.

Exercises to be handed in: (all exercise numbers refer to Dummit and Foote, 3rd edition.)

Section 5.5: **6, 8, 11.** Also, for problems 8 and 11, find presentations for all of the groups in your classification (but you needn't carefully prove that your presentations are correct.)

Section 6.1: **7, 24, 25, 26(a, b)** (I'm not sure I see the point of stating the result of (b) in terms of a universal property, so you can omit that.)

Exercise not from the text

1. Prove that if G is a group with $|G| < 60$, then G is solvable. Show that if $|G| = 60$ and G is not solvable, then $G \cong A_5$. (You may quote the result of Proposition 21 in the text.) Thus A_5 is the unique smallest non-solvable group.

(Hint: First show that it is enough to prove that every G with order < 60 is not simple. Then use the Sylow theorems and other techniques we have for finding normal subgroups. Most values of $|G|$ will fall into a case we or the book already considered, but a few will require some work.)