

Math 200a Fall 2012 Homework 6

Due *Wednesday* 11/21/2012 *in class*.

Upcoming schedule: Note that this homework is due Thanksgiving week. Class will meet the Wednesday of Thanksgiving week as usual, but you are welcome to hand in the homework earlier if you will not be here. Note that the homework boxes in the basement will be unavailable that week due to construction work, so you must get your homework directly to me or Johanna.

Reading assignment: Chapter 7.

Exercises to be handed in: (all exercise numbers refer to Dummit and Foote, 3rd edition.)

Section 7.1: **14, 26, 27**

Section 7.2: **3(b, c)**

Section 7.3: **29, 33**

(Remarks: For #29, use the binomial theorem which is stated in exercise #25.

For #33(a), the hard part is to show that if $\sum a_i x^i$ is a unit, then a_i is nilpotent for $i \geq 1$. There are direct ways of doing this by playing around, but here is a hint for a slick way of doing it. Prove each a_i with $i \geq 1$ is nilpotent by showing that it is in every prime ideal, and using the theorem which we will prove in class that the nilradical of a commutative ring is precisely the intersection of all of the prime ideals of the ring. To prove that each a_i with $i \geq 1$ is in any particular prime ideal P of R , consider the ring homomorphism $R[x] \mapsto (R/P)[x]$.)

Section 7.4: **35, 36, 39, 40**

(Remark: For 7.4 #36 you need to show some poset has a minimal element. The trick for doing this is to apply Zorn's lemma to the poset which is the opposite of the obvious one, i.e. for prime ideals you define $P_1 \leq P_2$ if $P_2 \subseteq P_1$. Then a maximal element in this ordering is a prime minimal under inclusion.)