MATH 200A FALL 2021 MIDTERM

Instructions:

You may quote major theorems proved in class or the notes, but not if the whole point of the problem is to reproduce the proof of such a theorem. Similarly, quote the result of a homework exercise only if the result of the exercise is a fundamental fact and reproducing the result of the exercise is not the main point of the problem.

You may use the result of one part of the problem in the proof of a later part, even you do not complete the earlier part.

1 (15 pts).

(a) If P is a Sylow p-subgroup of a finite group G, prove that $N_G(N_G(P)) = N_G(P)$. (This was a homework exercise— I want you to reprove it).

(b) Let p be an odd prime. Prove that a group G of order $4p$ has a normal Sylow p-subgroup if and only if G has a subgroup of order $2p$.

(c) Show that A_4 has no subgroup of order 6.

2 (15 pts).

(a) Consider the group with presentation $(x, y | x^2 = 1, y^2 = 1, (xy)^n = 1)$ for some $n \geq 3$. Show that this group is isomorphic to the dihedral group D_{2n}.

(b) Suppose that G is any finite group which has order at least 6 and is generated by two elements of order 2. Show that $G \cong D_{2n}$ for some $n \geq 3$.

Date: November 1, 2021.