MATH 200B WINTER 2021 MIDTERM

Instructions: As previously announced, this is an open book exam— you may use Isaacs, Dummit and Foote, and any class materials if you wish (lectures, homework writeups, course notes). You may not refer to any other textbooks or online sources.

You may make use of theorems proved in class, the notes, or one of the approved textbooks, but not if it trivializes the problem. Avoid quoting the results of homework exercises.

This is a 50 minute exam plus 10 minutes for downloading and 15 minutes for uploading. All problems are worth 10 points. Your exam must be uploaded by 12:05pm Pacific time (or if you are taking the exam at another time, at the analogous time). Since there may be other students taking this exam at different times, you must keep the exam problems confidential. Do not discuss the exam problems with anyone else until the day after your exam.

1. Let R be an integral domain. Recall that a left R-module M is called *divisible* if for all $x \in M$, and $0 \neq r \in R$, there exists $y \in M$ such that $ry = x$.

(a). Let M be any left R-module and let N be a torsion left R-module. Prove that $M \otimes_R N$ is again a torsion left R-module.

(b). Let M be a divisible left R-module and again let N be a torsion left R-module. Prove that $M \otimes_R N = 0$.

2. Let R be a PID. Suppose that there exists a nonzero finitely generated divisible R-module M. Prove that R is a field.

3. A matrix $A \in M_2(F)$ has a *square root* if there is $B \in M_2(F)$ such that $B^2 = A$.

Let F be an algebraically closed field of characteristic 2. Which matrices $A \in M_2(F)$ have a square root?

Date: February 8, 2021.