Homework 4 answers to part of problems

Problem 2.1.12. [Graph omitted]

Problem 2.1.14.

Problem 2.1.16.
2.2.4

(a) \( \lim_{(x,y) \to (0,1)} (e^x y) = e^0 \times 1 = 1 \)

(b) \( \lim_{x \to 0} \frac{\sin(x)}{x} \times \lim_{x \to 0} x = 1 \times 0 = 0 \)

Note: \( \lim_{x \to 0} \frac{\sin(x)}{x} = 1 \)

(c) \( \lim_{x \to 0} \frac{\sin^2(x)}{x^2} = 1^2 = 1 \)

2.2.6

(a) \( \lim_{(x,y) \to (0,0)} \frac{xy^3}{x^2 + y^6} = \lim_{(x,y) \to (0,0)} \frac{0 \times y^3}{0 + y^6} = \lim_{(x,y) \to (0,0)} \frac{0}{y^6} = 0 \)

(b) \( \lim_{(x,y) \to (0,0)} \frac{xy^3}{x^2 + y^6} = \lim_{(x,y) \to (0,0)} \frac{y^3 \times y^3}{y^2 + y^6} = \lim_{(x,y) \to (0,0)} \frac{y^6}{2y^6} = 0.5 \)

(c) Since (a) is not equal to (b), this function is not continuous at (0,0)

2.3.2

(a)
\[ \frac{\partial z}{\partial x}(0,0) = 0 \]
\[ \frac{\partial z}{\partial y}(0,0) = 0 \]
\[ \frac{\partial z}{\partial x}(\frac{a}{2},\frac{a}{2}) = -\frac{a}{|a|} \frac{\sqrt{2}}{2} \]
\[ \frac{\partial z}{\partial y}(\frac{a}{2},\frac{a}{2}) = -\frac{a}{|a|} \frac{\sqrt{2}}{2} \]

(b)

\[ \frac{\partial z}{\partial x} = \frac{y}{2(1 + xy)} \]
\[ \frac{\partial z}{\partial y} = \frac{x}{2(1 + xy)} \]
\[ \frac{\partial z}{\partial x}(0,0) = 0; \quad \frac{\partial z}{\partial y}(0,0) = 0 \]
\[ \frac{\partial z}{\partial x}(1,2) = \frac{1}{3}; \quad \frac{\partial z}{\partial y}(1,2) = \frac{1}{6} \]

(c)

\[ \frac{\partial z}{\partial x} = \alpha e^{ax} \cos(bx + y) - e^{ax} \sin(bx + y) \]
\[ \frac{\partial z}{\partial y} = -e^{ax} \sin(bx + y) \]
\[ \frac{\partial z}{\partial x}(\frac{2\pi}{b},0) = \alpha e^{\alpha x} = \alpha e^{\frac{\pi}{a}} \]
\[ \frac{\partial z}{\partial y}(\frac{2\pi}{b},0) = 0 \]

2.3.6

\[ Z = f(x_0, y_0) + \frac{\partial f}{\partial x}(x_0, y_0) \times (x - x_0) + \frac{\partial f}{\partial y}(x_0, y_0) \times (y - y_0) = 1 + x + y \]
Equation of plane: \[ x + y - z + 1 = 0 \]

2.3.12

(a)

\[ \frac{\partial f}{\partial x} = 2e^{(2x+3y)} \]
\[ \frac{\partial f}{\partial y} = 3e^{2x+3y} \]
\[ z = 1 + 2x + 3y \rightarrow \text{Equation of plane: } 2x + 3y - z + 1 = 0 \]
(b) approximation

\[ f(0.1, 0) = 1.2 \]
\[ f(0, 0.1) = 1.3 \]

(c) use calculator to get answer

2.3.24

\[
\frac{\partial h}{\partial x} = e^{(x-y)} + (x + z) \times e^{(x-y)}
\]

\[
\frac{\partial h}{\partial y} = -(x + z)e^{x-y}
\]

\[
\frac{\partial h}{\partial z} = e^{x-y}
\]

Answer: (3, -2, 1)