Math 20C - Fall 2020
Calculus & Analytic Geometry

(The goal of this class is to develop Calculus in 2D & 3D.)

A few course details.

- LINKS ARE ON THE CANVAS PAGE
- COURSE WEBSITE: schedule, contact info, acts as a syllabus.
- DISCORD SITE: Good place to ask questions & find study partners.
• CANVAS/GRADESCOPES
• WEBASSIGN: Our HW site. Link on Canvas in the Modules tab.
• ZOOM: Where lectures happen (recordings are linked to on CANVAS)

The course grade has 2 components
• quiz scores
• homework scores

FINAL GRADE = 60% (quiz scores) + 40% (homework scores)

QUESTIONS? Unmute & ASK! (or ask in chat)
§1.1 Vectors in 2D & 3D.

This pair of real numbers \((a, b)\) makes a vector starting at \((0,0)\) and going to \((a, b)\).
IN 3D, a similar picture

\[ (a, b, c) = q \]

SOME NOTATION.

The set of REAL NUMBERS is denoted \( \mathbb{R} \).

The set of PAIRS OF REAL NUMBERS aka 2D VECTORS is denoted by \( \mathbb{R}^2 \).
TRIPLES OF REAL #s aka 3D vectors are denoted by $\mathbb{R}^3$.

**Adding Vectors**

1. Geometrically
   
   (similar picture in 3D)

2. The formulas

   \[ \vec{w} = (4, 2, 3) \]
   \[ \vec{v} = (2, 1, 8) \]
   \[ \vec{v} + \vec{w} = (3, 3, 11) \]
(So you add each coordinate ...) 

COMMENT: CAN'T ADD 2D + 3D vector. 

**SCALAR MULTIPLICATION**

Start with \( \vec{v} \in \mathbb{R}^2 \) (or \( \vec{w} \in \mathbb{R}^3 \))

(Read this as "\( \vec{v} \) in \( \mathbb{R}^2 \)"

AND \( x \in \mathbb{R} \) a real #.)
We can multiply!

\[ \lambda \cdot \mathbf{v} \]  (or \( \lambda \mathbf{v} \))

(called scalar multiplication)

1. The Formula:

If \( \mathbf{v} = (a, b, c) \) then

\[ \lambda \mathbf{v} = (\lambda a, \lambda b, \lambda c) \]

(multiply each coordinate)
20. Geometrically

If \( \lambda > 0 \)

\( \lambda \mathbf{v} \) bigger!
(Same direction)

If \( \lambda < 0 \)

\( \lambda \mathbf{v} \) same direction
Exact opposite direction
If \( x = 0 \) \( \Rightarrow x + (0,0) = \overrightarrow{0} \),

"the zero vector"

(SIMILAR in 3D).

Lines in 2D & 3D

Need: 1. A point.
   2. A direction vector.
PARAMETRIC EQN FOR A LINE

\[ \mathbf{v} = (c, d) \]
\[ P = (a, b) \]

Every point on \( L \) is given by

1. Taking \( P \)
2. Adding a vector in the direction of \( \mathbf{v} \)
   (or the opposite direction)

So: Every point on \( L \) has the form: \[ P + t \cdot \mathbf{v} \]
for some real $\neq t$.

So $L$ is "parametrized" by function:

$$\vec{f}(t) = (a, b) + t \cdot (c, d)$$
$$= (a + tc, b + td)$$

\[\underline{Example}\]. The parametric equation for the line through $(1,1,4)$ in the direction of $(0,3,5)$ is
\[ f(t) = (1,1,4) + t \cdot (0,3,5) \]
\[ = (1,1+3t, 4+5t) \]

**Exercise (in Breakout)**

(*) Suppose that in the previous example \( t \) represents "time". Does the point \((1,-5, -6)\) lie on that line? At what time?
(#2) Find the parametric equations for the lines:

L_1: through (0,0,1) in the direction of (-1,1,3).

L_2: through the point (2,1,6) in the direction of (3,-2,2).

(#3) Finally, find where L_1 \& L_2 intersect.