Wednesday, October 7

(PRESS RECORD...)

Questions?

Last time: Introduced the INNER PRODUCT (AKA Dot Product) of vectors...

- If \( \vec{V} = (X_1, Y_1, Z_1) \)
  & \( \vec{W} = (X_2, Y_2, Z_2) \)
  then \( \vec{V} \cdot \vec{W} = X_1X_2 + Y_1Y_2 + Z_1Z_2. \)

- Geometric Meaning.
  
  A. \( \| \vec{V} \| = \sqrt{\vec{V} \cdot \vec{V}} \)  
     length of \( \vec{V} \)

  B. If \( \vec{V} \) & \( \vec{W} \) point in the SAME direction then
     \( \vec{V} \cdot \vec{W} = (\text{length of } \vec{V}) (\text{length of } \vec{W}) \)
C. Theorem For \( \vec{v}, \vec{w} \in \mathbb{R}^2 \) or \( \mathbb{R}^3 \):

Any vectors...

\[ \vec{v} \cdot \vec{w} = \|\vec{v}\| \|\vec{w}\| \cos \theta. \]

Consequence. \( \vec{v} \perp \vec{w} \) are perpendicular (also called orthogonal) \( \iff \vec{v} \cdot \vec{w} = 0 \).

Exercise. (A). What is the angle between \( (1,1,0) \) and \( (0,1,1) \)?

(B). Which vectors are orthogonal?

\( (1,0,1), (1,1,1), (0,2,2) \)

(A). \( (1,1,0) \cdot (0,1,1) = 1 \).

\[ \| (1,1,0) \| = \sqrt{2} \quad \| (0,1,1) \| = \sqrt{2}. \]

\[ 1 = 2 \cos \theta. \]

\[ \frac{1}{2} = \cos \theta. \]

\[ \theta = \pi/6. \]
(B). \((4,0,1) \cdot (1,1,1) = 0.\)
\((0,1,1) \cdot (0,2,2) = -2.\)
\((1,1,1) \cdot (0,2,2) = 4.\)

Some notation:

The vectors \(\mathbf{i}, \mathbf{j}, \mathbf{k}\)...

In 2D
\[
\mathbf{i} \text{ (or } \hat{\mathbf{i}} \text{)} = (1,0) \\
\mathbf{j} \text{ (or } \hat{\mathbf{j}} \text{)} = (0,1)
\]

In 3D
\[
\mathbf{i} \cdot \mathbf{j} = (1,0,0) \\
\mathbf{j} \cdot \mathbf{k} = (0,1,0) \\
\mathbf{k} \cdot \mathbf{e} = (0,0,1)
\]

\(\mathbf{i}, \mathbf{j}, \mathbf{k}\) are all orthogonal with length 1.

When \(\|\mathbf{v}\| = 1\) we say \(\mathbf{v}\) is a normal vector or a unit vector.
For any nonzero vector $\vec{v}$ there is 1 unit vector $\vec{n}$ in the same direction.

Going from $\vec{v}$ to $\vec{n}$ we say we have normalized $\vec{v}$.

**Formula:** $\vec{n} = \frac{1}{\|\vec{v}\|} \cdot \vec{v}$

**Remark:** Formulas with normal vectors are often simpler, e.g.: $\vec{n}_1 \cdot \vec{n}_2 = \cos(\theta)$
Example. Normalizing \((1,2,3)\)

given the vector:

\[
\hat{v} = \frac{1}{\sqrt{1+4+9}} \cdot (1,2,3) = \left( \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \right).
\]

The ORTHOGONAL PROJECTION

Start with \(\vec{w} \neq \vec{0}\).
The orthogonal projection of \( F \) onto \( \overrightarrow{w} \) is the vector \( \overrightarrow{p} \).

So that \( F - \overrightarrow{p} \) is orthogonal to \( \overrightarrow{w} \).

I.e. It is the "part of \( \overrightarrow{v} \)" pointing in the \( \overrightarrow{w} \)-direction (or the opposite direction).

Q. How can we solve for \( \overrightarrow{p} \)?

\[ \overrightarrow{v} = \overrightarrow{p} + \overrightarrow{w}. \]

1. \( \overrightarrow{p} = \lambda \cdot \overrightarrow{w} \)
2. \( \overrightarrow{w} \cdot \overrightarrow{w} = 0 \)

Goal: Find \( \lambda \).

Know: \( (\overrightarrow{v} - \lambda \overrightarrow{w}) \cdot \overrightarrow{w} = 0 \).

So: \( \overrightarrow{v} \cdot \overrightarrow{w} - \lambda \overrightarrow{w} \cdot \overrightarrow{w} = 0 \).

We can solve: \( \lambda = \frac{\overrightarrow{v} \cdot \overrightarrow{w}}{\overrightarrow{w} \cdot \overrightarrow{w}} \).
Formula: $P = \left( \frac{\langle w, w \rangle}{\|w\|^2} \right) w$.

(can weirdly remember this as "the is cancel", this is entirely inappropriate.)

Example: What is the orthogonal projector of $(1, 1, 5)$ onto $(1, 1, 1)$?

$$P = \left( \frac{(1, 1, 5) \cdot (1, 1, 1)}{(1, 1, 1) \cdot (1, 1, 1)} \right) (1, 1, 1)$$

real #

$$= \frac{7}{3} (1, 1, 1) = \left( \frac{7}{3}, \frac{7}{3}, \frac{7}{3} \right)$$

Practice Quiz Time.
1. Go to Canvas & go to quizzes on the left.
2. It should be there?