HOMEWORK 1 - MATH 100A - DUE WEDNESDAY OCTOBER 4TH

Problem 1. (From Herstein §1.2 #5) If $A \subset B$ and $B \subset C$, prove that $A \subset C$.

Problem 2. (From Herstein §1.2 #8) Prove that $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$. What does this look like pictorially?

Problem 3. (From Herstein §1.2 #14) If C is a finite set, let $m(C)$ denote the number of elements in C. If A, B are finite sets, prove that

$$m(A \cup B) = m(A) + m(B) - m(A \cap B).$$

Problem 4. (From Herstein §1.3 #1) If $s_1 \neq s_2$ are in S, show that there is an $f \in A(S)$ such that $f(s_1) = s_2$.

Problem 5. (From Herstein §1.3 # 13) Show that there is a positive integer t such that $f^t = i$ for all $f \in S_n$.

Problem 6. (From Herstein §1.5 # 12) Starting with 2, 3, 5, 7, ..., construct the positive integers $1+2\cdot3$, $1+2\cdot3\cdot5$, $1+2\cdot3\cdot5\cdot7$, ... Do you always get a prime number this way?

Problem 7. (From Herstein §1.6 #13) Prove by induction on n, that $n^3 - n$ is always divisible by 3.

Problem 8. (From Herstein §1.7 # 7) Verify the commutative law of multiplication $zw = wz$ in $\mathbb{C}$.

Problem 9. (From Herstein §1.7 # 17) Viewing the x-y plane as the set of all complex numbers $x + iy$, show that multiplication by i induces a $90°$ rotation of the $x - y$ plane in a counter-clockwise direction.

Problem 10. We say a complex number $\zeta \in \mathbb{C}$ is an n-th root of unity if $\zeta^n = 1$ (equivaleintly if ζ is a root of $x^n - 1$). Prove that the sum of all n-th roots of unity is 0.