1. Define $\phi : B^A \rightarrow P(A)$ by $\phi(f) = \{x \in A | f(x) = 0\}$. We will show that $\phi$ is a bijection. Suppose that $f_1, f_2 \in B^A$ and $\phi(f_1) = \phi(f_2)$, i.e. $\{x \in A | f_1(x) = 0\} = \{x \in A | f_2(x) = 0\}$. Let $y \in A$. If $f_1(y) = 0$, then $y \in \{x \in A | f_1(x) = 0\} = \{x \in A | f_2(x) = 0\}$, which implies that $f_2(y) = 0$. If $f_1(y) = 1$, then $y \notin \{x \in A | f_1(x) = 0\} = \{x \in A | f_2(x) = 0\}$, which implies that $f_2(y) \neq 0$. But since $B = \{0, 1\}$ is the codomain of $f_2$, this implies that $f_2(y) = 1 = f_1(y)$. Hence, $f_1(y) = f_2(y)$ in both cases. It follows that $f_1 = f_2$, so $\phi$ is injective.

Now, let $S \in P(A)$, so $S \subseteq A$. Define a function $f : A \rightarrow B$ by

$$f(x) = \begin{cases} 0 & \text{if } x \in S \\ 1 & \text{if } x \notin S. \end{cases}$$

Then from the definition of $f$, we see that

$$\phi(f) = \{x \in A | f(x) = 0\} = S.$$ 

Thus, $\phi$ is surjective and is therefore a bijection.

3. This is not an equivalence relation since it is not symmetric - $1 \geq 0$ but $0 \not\geq 1$.

8. $10 + 17 = 16 = 9$
    $8 + 10 = 6 = 4$
    $20.5 + 25 = 19.3 = 14.8$
    $1/2 + 7/8 = 3/8$

9. Note that

$$e^{i a} e^{i b} = e^{i(a + b)} = \cos(a + b) + i \sin(a + b) \tag{1}$$

By Euler’s formula. On the other hand,

$$e^{i a} e^{i b} = (\cos a + i \sin a)(\cos b + i \sin b) = (\cos a \cos b - \sin a \sin b) + i(\cos a \sin b + \sin a \cos b). \tag{2}$$

Equating real and imaginary parts of (1) and (2), we see that

$$\cos(a + b) = (\cos a \cos b - \sin a \sin b)$$
$$\sin(a + b) = (\cos a \sin b + \sin a \cos b).$$