Problem 1. (§0 #18) Let $A$ be any set (possibly infinite) and let $B = \{0, 1\}$. Define $B^A$ to be the set of all functions mapping $A$ into the set $B = \{0, 1\}$. Show that the cardinality of $B^A$ is the same as the cardinality of the power set $P(A)$.

In problems 2-4, either prove $R$ is an equivalence relation, or say why it isn’t.

Problem 2. (§0 #29) $nRm$ in $\mathbb{Z}$ if $n \cdot m > 0$.

Problem 3. (§0 #30) $xRy$ in $\mathbb{R}$ if $x \geq y$.

Problem 4. (§0 #31) $xRy$ in $\mathbb{R}$ if $|x| = |y|$.

Problem 5. Which of the following numbers are congruent modulo 8:

- 2, 6, 10, 14, 18, 81, 162.

Problem 6. (§1 #2,5) Write the following complex numbers as $a + bi$ where $a, b \in \mathbb{R}$.

1. $i^4$
2. $(4 - i)(5 + 3i)$

Problem 7. (§1 #14, 15) Write the following complex numbers $z$ in polar form. I.e. write $z$ as $|z|(p + qi)$ where $p, q \in \mathbb{R}$ and $|p + qi| = 1$.

1. $z = 12 + 5i$
2. $z = -3 + 5i$

Problem 8. (§1 #22 - 25) Compute the following expressions using modular addition.

1. $10 +_{17} 16$
2. $8 +_{10} 6$
3. $20.5 +_{25} 19.3$
4. $1/2 +_{1} 7/8$

Problem 9. (§1 #38) Derive the formulas

$$\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

and

$$\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

by using Euler’s formula and computing $e^{ia}e^{ib}$.

Problem 10. Let $n \geq 2$ be an integer. Prove that the sum of all the $n$th roots of unity is 0. I.e. show

$$\sum_{z \in U_n} z = 0.$$