Problem 1. (§2 #7) Let \((S, \ast)\) and \((T, \ast')\) be 2 binary structures. Prove that if 
\[\phi: S \rightarrow T\]
is an isomorphism of binary structures then so is \(\phi^{-1}\).

Problem 2. Is \(7\mathbb{Z}\) a subgroup of \((\mathbb{C}, \cdot)\)? If yes, prove it. If no, say why not.

Problem 3. Is the set \(\{\pi^n | n \in \mathbb{Z}\}\) a subgroup of \((\mathbb{C}, \cdot)\)? If yes prove it. If no, say why not.

Problem 4. Is the subset 
\[\{A \in \text{GL}_n(\mathbb{R}) | A \cdot A^t = \text{Id}_n\} \subset \text{GL}_n(\mathbb{R})\]
a subgroup of \(\text{GL}_n(\mathbb{R})\)? (Here \(A^t\) is the transpose of the matrix \(A\).) If yes, prove it. If no, say why not.

Problem 5. In the following examples find the order of the cyclic subgroup generated by 
the given element.

1. The subgroup of \(\mathbb{Z}_4\) generated by 3.
2. The subgroup of \(U_6\) generated by \(\cos(2\pi/3) + i\sin(2\pi/3)\).
3. The subgroup of \(\mathbb{C}^*\) generated by \(1 + i\).
4. The subgroup of \(\mathbb{C}^*\) generated by \((1 + i)/\sqrt{2}\).

Problem 6. Let \(G\) and \(H\) be two isomorphic groups. Prove that if \(G\) is cyclic, then \(G'\) is also cyclic.

Problem 7. In the following examples, find all the subgroups of the given groups.

1. \(\mathbb{Z}_{12}\)
2. \(\mathbb{Z}_{36}\)
3. \(\mathbb{Z}_{17}\)

Problem 8. Let \(a\) and \(b\) be elements of a group \(G\). Show that if \(ab\) has finite order \(n\), then \(ba\) also has order \(n\).

Problem 9. Let \(G\) be a group. Let \(H, K \leq G\) be two subgroups of \(G\). Prove that the subset \(H \cap K \subset G\) is a subgroup of \(G\).

Problem 10. Let \(p\) and \(q\) be distinct prime numbers. Find the number of generators of 
the cyclic group \(\mathbb{Z}_{pq}\).