Problem 1. Consider the following element $\sigma = (145) \circ (78) \circ (257) \in S_8$. Write $\sigma$ in cycle notation.

Problem 2. Using $\sigma$ from the previous problem, write down the associated permutation matrix, $A_\sigma \in \text{GL}_n(\mathbb{R})$.

Problem 3. Find the maximum possible order for an element of $S_5$ and $S_6$.

Problem 4. Find all the cosets of the subgroup $4\mathbb{Z} \leq \mathbb{Z}$.

Problem 5. Find all the cosets of the subgroup $4\mathbb{Z} \leq 2\mathbb{Z}$.

Problem 6. Find the index of $\langle 3 \rangle$ in the group $\mathbb{Z}_{24}$.

Problem 7. Find the index of the subgroup $U_4 \leq U_{40}$.

Problem 8. Let $G$ be a group of order $pq$, where $p$ and $q$ are prime numbers. Prove that every proper subgroup of $G$ is cyclic.

Problem 9. Let $G$ be a finite group of order $n$. Prove that for all $x \in G$, $x^n = e$.

Problem 10. If $G$ is a finite group, define the exponent of $G$, to be the number

$$\exp(G) = \min\{k > 0 | x^k = e \text{ for all } x \in G\}.$$ 

Give an example of a finite group such that $\exp(G) \neq |G|$. 