Problem 1. Let $G$ be an abelian group, show that the “inverse function"

$$\text{inv} : G \to G, \quad x \mapsto x^{-1}$$

is a homomorphism.

Problem 2. Suppose $G$ is not an abelian group, show that the inverse function from Problem 1 is not a homomorphism.

Problem 3. Show that the function

$$| - | : \mathbb{R}^* \to \mathbb{R}^*, \quad x \mapsto |x|$$

is a homomorphism.

Problem 4. Using the homomorphism from the previous problem, find $\text{Ker}(| - |)$ and $\text{Im}(| - |)$.

Problem 5. Let $G$ be a group. Show that the center of $G$, $Z(G)$, is a normal subgroup.

Problem 6. Show that the subgroup of rotations $\langle \rho \rangle \leq D_6$ is a normal subgroup.

Problem 7. How many homomorphisms are there $\phi : \mathbb{Z} \to \mathbb{Z}_6$.

Problem 8. Suppose that $G$ is a group and $|G| = p$ is a prime number. Show that any homomorphism:

$$\phi : G \to H$$

either satisfies $\text{Im}(\phi) = \{e\}$ or $\phi$ is injective.

Problem 9. Suppose $\phi : G \to H$ is a group homomorphism, and let $K \leq H$ be a subgroup. Show that the set

$$\phi^{-1}(K) := \{ x \in G | \phi(x) \in K \}$$
is a subgroup of $G$.

Problem 10. Let $\phi : G \to H$ be a homomorphism. Show that $\text{Im}(\phi)$ is abelian if and only if $x * y * x^{-1} * y^{-1} \in \text{Ker}(\phi)$ for all $x, y \in G$. 