Problem 1. Use the Fundamental Homomorphism Theorem to prove that the quotient group $\mathbb{Z}/2\mathbb{Z}$ is isomorphic to $\mathbb{Z}_2$.

Problem 2. Use the Fundamental Homomorphism Theorem to prove that the quotient group $\mathbb{Z}/n\mathbb{Z}$ is isomorphic to $\mathbb{Z}_n$.

Problem 3. Let $\phi: G \to H$ be a surjective group homomorphism. Prove that if $G$ is a cyclic group, then $H$ is a cyclic group.

Problem 4. Let $\phi: G \to H$ be a surjective group homomorphism. Prove that if $G$ is an abelian group, then $H$ is an abelian group.

Problem 5. Let $G$ be an abelian group. Prove that every subgroup $H \leq G$ is a normal subgroup.

Problem 6. Show that there is only one homomorphism from $D_9$ to $\mathbb{Z}_5$.

Problem 7. Show that there is only one homomorphism from $S_5$ to $\mathbb{Z}_7$.

Problem 8. Let $\phi: G \to H$ be a group homomorphism, and let $K \leq H$ be a subgroup. In Problem 9 of the previous homework we defined $\phi^{-1}(K)$ and showed that $\phi^{-1}(K)$ is a subgroup of $G$. If $K$ is a normal subgroup of $H$, prove that $\phi^{-1}(K)$ is a normal subgroup of $G$.

Problem 9. Recall that $S^1 = \{z \in \mathbb{C}|z \text{ has distance one from } 0\} \leq \mathbb{C}^*$ is the group of complex numbers with radius 1 with the operation given by complex multiplication. Show that the following function is a group homomorphism:

$$
\psi: \mathbb{R} \to S^1
$$

$$
a \mapsto z = \psi(a)
$$

(where $z$ is the complex number with radius 1 and angle $2\pi a$).

Problem 10. Using the Fundamental Homomorphism Theorem and the homomorphism $\psi$ from the Problem 9, prove that $S^1 \cong \mathbb{R}/\mathbb{Z}$. 