Instructions:

• Do not open the exam until you are instructed to do so.

• Write your name and student ID number on the front page of the exam.

• Write your name and student ID number at the top of every page of the exam.

• Answer the questions in the spaces provided. If you run out of room for an answer, continue on the back of the page.

• You must show your work!

• If you need more paper, ask one of the proctors and we will provide it.

• There is an extra page at the end of the exam for scratch work.

Math 20E - Midterm 1 - 1/29/2020

Name & Student ID: ____________________________________________________________

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1. (a) (3 points) Let \( R \) be the region \([0, 3] \times [2, 4] \). Evaluate the integral

\[
\int \int_{R} e^{2x-3y} \, dy \, dx.
\]

**Answer:** \( \frac{(e^6 - 1)^2}{6e^6} \)

(b) (3 points) Let \( D \) be the region in \( \mathbb{R}^2 \) between the curves \( y = x^2 \) and \( y = x^3 \). Evaluate the integral

\[
\int \int_{D} xy \, dy \, dx.
\]

**Answer:** \( \frac{1}{2} \left( \frac{1}{6} - \frac{1}{8} \right) \)
2. (4 points) Change the order of integration and evaluate:

\[ \int_0^1 \int_{\sqrt{y}}^1 e^{x^3} \, dx \, dy. \]

**Answer:** \( \frac{1}{3}(e - 1). \)
3. (a) (1 point) Let $R$ be the region in $\mathbb{R}^2$ given by the three equations

$$x^2 + y^2 \leq 2, \ x \geq 0, \ and \ y \geq 0.$$

Describe $R$ in words. (What shape is $R$?)

**Answer:** $R$ is a quarter circle of radius $\sqrt{2}$ in the 1st quadrant.

(b) (2 points) Set up an integral in $(x,y)$-coordinates which computes the area of this region. You do not need to evaluate this integral.

**Answer:**

$$\int_{0}^{\sqrt{2}} \int_{0}^{\sqrt{2-x^2}} 1 \, dy \, dx.$$ 

(c) (2 points) Set up an integral in polar coordinates which computes the area of this region. You do not need to evaluate this integral.

**Answer:**

$$\int_{0}^{\pi/2} \int_{0}^{\sqrt{2}} r \, dr \, d\theta.$$
4. (a) (1 point) Let:

\[ T : \mathbb{R}^2 \to \mathbb{R}^2 \]

be a differentiable function such that \( T(u, v) = (x(u, v), y(u, v)) \). What is the Jacobian determinant \( \frac{\partial (x,y)}{\partial (u,v)} \)?

**Answer:**

\[
\frac{\partial (x,y)}{\partial (u,v)} = \\
\begin{vmatrix}
\frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\
\frac{\partial y}{\partial u} & \frac{\partial y}{\partial v}
\end{vmatrix}
\]

(b) (2 points) Let \( D \) be the region in the the first quadrant of the plane \( (x \geq 0 \text{ and } y \geq 0) \) where \( 3 \leq \sqrt{x^2 + y^2} \leq 4 \). If \( f(x,y) \) is a function, set up an integral in polar coordinates which computes

\[
\iint_D f(x,y) \, dy \, dx.
\]

**Answer:**

\[
\pi/2 \int_0^4 \int_3 f(r \cos(\theta), r \sin(\theta)) r \, dr \, d\theta.
\]

(c) (3 points) Let \( D \) be the region from part (b). Use polar coordinates to calculate:

\[
\iint_D \frac{1}{x^2 + y^2} \, dy \, dx.
\]

**Answer:**

\[
\frac{\pi}{2} (\log(4) - \log(3)).
\]
Scratch Paper.