Instructions:

- Do not open the exam until you are instructed to do so.
- Write your name and student ID number on the front page of the exam.
- Write your name and student ID number at the top of every page of the exam.
- Answer the questions in the spaces provided. If you run out of room for an answer, continue on the back of the page.
- You must show your work!
- If you need more paper, ask one of the proctors and we will provide it.
- There is an extra page at the end of the exam for scratch work.

Math 20E - Midterm 1 - 1/29/2020

Name & Student ID: _____

Question:	1	2	3	4	Total
Points:	6	4	5	6	21
Score:					

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1. (a) (3 points) Let R be the region $[0,3] \times [2,4]$. Evaluate the integral

$$\iint_R e^{2x-3y}\,dy\,dx.$$

Answer: $\frac{(e^6-1)^2}{6e^{12}}$

(b) (3 points) Let D be the region in $\mathbb{R}^2$ between the curves $y = x^2$ and $y = x^3$. Evaluate the integral

$$\iint_D xy\,dy\,dx.$$

Answer: $\frac{1}{2}(\frac{1}{6} - \frac{1}{8})$

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2. (4 points) Change the order of integration and evaluate:

$$\int_0^1 \int_{\sqrt{y}}^1 e^{x^3}\, dx\, dy.$$

Answer: $\frac{1}{3}(e-1)$.

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3. (a) (1 point) Let R be the region in $\mathbb{R}^2$ given by the three equations

$$x^2 + y^2 \leq 2,\ x \geq 0, \text{ and } y \geq 0.$$

Describe R in words. (What shape is R?)
Answer: R is a quarter circle of radius $\sqrt{2}$ in the 1st quadrant.

(b) (2 points) Set up an integral in (x, y)-coordinates which computes the area of this region. You do not need to evaluate this integral.
Answer: $\int_0^{\sqrt{2}} \int_0^{\sqrt{2-x^2}} 1 dy dx$.

(c) (2 points) Set up an integral in polar coordinates which computes the area of this region. You do not need to evaluate this integral.
Answer: $\int_0^{\pi/2} \int_0^{\sqrt{2}} r dr d\theta$.

4. (a) (1 point) Let:

$$T : \mathbb{R}^2 \to \mathbb{R}^2$$

be a differentiable function such that $T(u, v) = (x(u, v), y(u, v))$. What is the *Jacobian determinant* $\frac{\partial(x,y)}{\partial(u,v)}$?

Answer: $\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$.

(b) (2 points) Let D be the region in the the first quadrant of the plane ($x \geq 0$ and $y \geq 0$) where $3 \leq \sqrt{x^2 + y^2} \leq 4$. If $f(x, y)$ is a function, set up an integral in polar coordinates which computes

$$\iint_D f(x, y)\,dy\,dx.$$

Answer: $\int_0^{\pi/2} \int_3^4 f(r\cos(\theta), r\sin(\theta))\,r\,dr\,d\theta$.

(c) (3 points) Let D be the region from part (b). Use polar coordinates to calculate:

$$\iint_D \frac{1}{x^2 + y^2}\,dy\,dx.$$

Answer: $\frac{\pi}{2}(\log(4) - \log(3))$.

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Scratch Paper.