PRACTICE FINAL – MATH 20E – WINTER 2020

Problem #1. Evaluate

$$\iint_R \frac{yx}{y^2+2}\,dy\,dx$$

where $R = [0,2] \times [-1,1]$.

Problem #2. Integrate $f(x,y,z) = ze^{x^2+y^2}$ over the cylinder S defined by $x^2+y^2 \leq 4$ and $2 \leq z \leq 3$.

Problem #3. Find the volume of the region V given by $z \leq 6 - x^2 - y^2$ and $z \geq \sqrt{x^2+y^2}$.

Problem #4. Evaluate

$$\int_{\overline{c}} x\cos(z)\,ds$$

where $\overline{c}(t) = (t, t^2, 0)$ and $t \in [0,1]$.

Problem #5. Evaluate

$$\int_C y^2\,dx + 2xy\,dy$$

where C is the unit circle oriented counterclockwise.

Problem #6. Find a parametrization $\Phi\colon D \to \mathbb{R}^3$ of the surface $S \subset \mathbb{R}^3$ defined by:

$$x^2 + y^2/2 + z^2/3 = 1,$$

for some region $D \subset \mathbb{R}^2$.

Problem #7. Evaluate

$$\iint_S z\,dA$$

over the surface S defined by $x^2+y^2 \leq 1$ and $z = x^2+y^2$.

Problem #8. Evaluate

$$\iint_S 2x\,dx - 2y\,dy + z^2\,dz$$

where $S = x^2+y^2 = 4$ and $0 \leq z \leq 1$ is oriented outwards.

Problem #9. Use Green's theorem to evaluate:

$$\int_C x dx + y dy$$

where C is the boundary of the square $[-1, 1] \times [-1, 1]$ oriented counterclockwise.

Problem #10. Evaluate

$$\iint_S \nabla \times \mathbf{F} \cdot dA$$

using Stokes' theorem, where $\mathbf{F}(x, y, z) = (y, -x, zx^3y^2)$ and S is the surface defined by the equations: $x^2 + y^2 + z^2 = 1$, $z \leq 0$, and oriented upwards.

Problem #11. Let $V \subset \mathbb{R}^3$ be the region $x^2 + y^2 \leq z \leq 1$. Evaluate

$$\iint_{\partial V} (y, z, xz) \cdot dA$$

using Gauss's theorem.