Math 154, Spring 2021 HOMEWORK #1 due Thursday, April 8

Q1. (6pts) Answer parts (d), (e), and (f) of **Question 1.4** at the end of chapter 1 of the textbook. Explain how you know the answers are correct.

Q2. (10pts) The Handshake Lemma, as we have learned it, applies to simple graphs. Assume now that G is a digraph in which every edge is directed and loops are allowed, but there are no multiedges (there is at most one directed edge between any two vertices).

For this graph, the definitions of in- and out-neighborhoods and in- and out-degrees given in Lecture 2 apply exactly as given.

By modifying the proof of the Handshake Lemma to work for G, prove that

$$\sum_{v \in V} d_G^-(v) \; + \; \sum_{v \in V} d_G^+(v) \; = \; 2|E| \; .$$

Q3. (10pts) Answer all three parts of **Question 1.10** at the end of chapter 1 of the textbook.

Q4. (4pts) Let $n \geq 3$ be an integer. How many different Hamiltonian cycles that start and end at vertex 1 exist in K_n?