Math 154, Spring 2021 HOMEWORK #3 due Thursday, April 22

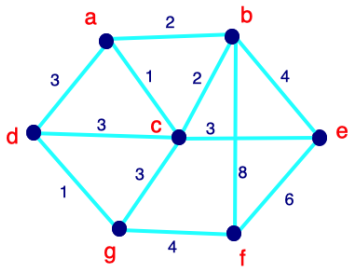
Q1. (12pts) Prove the following two facts about trees.

a) Any tree on $n \geq 2$ vertices has at least two leaves.

b) Any forest with k components (trees) on $n \geq k$ vertices has exactly $n - k$ edges.

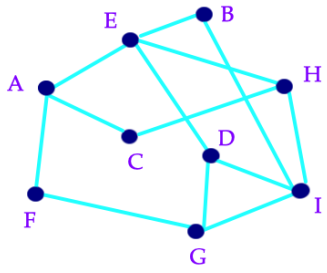
Q2. (12pts)

a) Consider the weighted graph below. Show how Prim's algorithm applied on it starting at the vertex a produces an MST, adding the edges in step-by-step fashion as demonstrated in lecture.



b) Do the same for Kruskal's algorithm for the graph above, making choices that preferably find a different MST than the one obtained with Prim's algorithm. (The graph has multiple MSTs).

c) Find a Breadth-First Search tree rooted at the vertex B for the graph below. Note that the vertices are labeled with letters; if at some point you have multiple vertices that you could add, pick the lowest one in alphabetical ordering first (e.g., if a vertex has available neighbors A, B, C, pick A first, then B, then C).



Q3. (6pts) Let G be a graph on n vertices which is disconnected. What is the maximum number of edges that G can have?