

Math 154, Spring 2021 HOMEWORK #6 due Thursday, May 20

Q1. (8pts) Solve Question 7.1 from the textbook, for the first two top graphs.

Q2. (14pts) Suppose G is a plane graph which contains no triangle as a subgraph, and for which the number of vertices is at least 3.

- a) (3pts) Show that for every face F , $\deg(F) \geq 4$.
- b) (4pts) Prove that $|E(G)| \leq 2|V(G)| - 4$.
- c) (3pts) Conclude that G is 3-degenerate, and therefore 4-colorable.
- d) (4pts) Give an example of a connected graph which satisfies $|E(G)| \leq 2|V(G)| - 4$ but it is not planar.

Q3. (8pts) Consider K_6 .

- a) Delete two of its edges. Show that the resulting graph is never planar (you will have to consider the cases when the two edges are disjoint, and when they share an endpoint).
- b) Is there a way to delete three edges so that the resulting graph is planar?