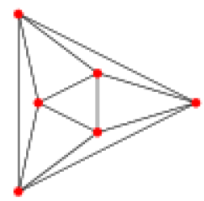
*Math 154, Spring 2021* *HOMEWORK #7* *due Thursday, May 27*

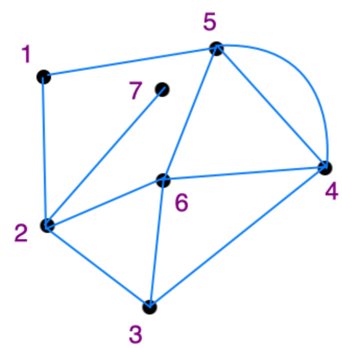
**Q1.** (10pts) A maximally planar graph on $n \geq 3$ vertices is a graph which has the property that the addition of any other edge would make it non-planar. An example of such a graph is the octahedron (below).



a) Show that in a maximally planar graph, every face is a triangle.

b) Conclude that in such a graph, $|E| = 3|V| - 6$ and $|F| = 2|V| - 4$.

**Q2.** (6pts) Draw the dual G' of the pseudo/multigraph G below, and state how many vertices, edges and faces each of the two pseudo/multigraphs have.




**Q3.** (8pts) Suppose $G$ is a maximally planar graph on $n \geq 3$ vertices. Show that if $G$ is 3-colorable, then $G$ has an Eulerian tour.

**Q4.** (6pts) Find three different feasible flows in the network represented in Figure 8.3 in the textbook. (Use just the picture; ignore Question 8.1).