Student name: _____

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MATH 10A (Butler)

Final, 19 March 2007

This test is closed book and closed notes, with the exception that you are allowed one $8\frac{1}{2}'' \times 11''$ page of handwritten notes. You may use any shortcuts for derivatives unless explicitly stated otherwise. No calculator is allowed for this test. For full credit show all of your work (legibly!), unless otherwise specified. You do not need to simplify your answers any more than the question requires. Any work that is done on scratch paper needs to be attached to the test and clearly stated in the problem where to find the work.

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1. (9 points) Find the derivative of the following function. (Hint: it might be useful to use properties of logarithms to simplify before taking the derivative.)

$$q(x) = \ln\left(\frac{739e^{5x}x^3}{(x^2+1)\cos(x)}\right)$$

2. (a) (2 points) Complete the following definition of the derivative for $f(x)$:

$$f'(x) = \lim_{h \to 0} \text{———————————}$$

(b) (8 points) Using the definition for the derivative given in part (a), find $f'(x)$ for $f(x) = \dfrac{1}{3 - 2x^2}$. [Remember to show all of your work!!]

3. Let $f(t) = te^{-t^2/6}$.

(a) (4 points) What are the critical points of $f(t)$?

(b) (4 points) Use either the first derivative test or the second derivative test to determine whether the critical points found in part (a) are local minimums, local maximums or neither. (Be sure to indicate which test you are using!!)

(Problem 3 continued)

(c) (4 points) Find all the inflection points of $f(t)$. [Hint: there are three of them.]

4. (MULTIPLE CHOICE QUESTIONS, 3 points each). Write your answer in the space provided. There is no partial credit for incorrect answers.

_____ If $g(t) = 3^t$ then the graph of $g(t+1)$ is
(a) a horizontal shift to the LEFT by 1 of the graph of $g(t)$.
(b) a horizontal shift to the RIGHT by 1 of the graph of $g(t)$.
(c) a vertical stretch by a factor of 2 of the graph of $g(t)$.
(d) two of the above.
(e) none of the above.

_____ You are given a function $f(v)$ which takes as its input a velocity (measured in miles per hour) and as an output gives a measurement of the fuel consumption of a car (measured in gallons per hour). If you were to explain the meaning of $f'(65) = 0.05$ to someone who has not had calculus you could tell them:
(a) When going at 65 miles per hour, the car is using 0.05 gallons per hour.
(b) When going at 65 miles per hour, the car is traveling 0.05 miles per gallon.
(c) When going at 65 miles per hour, the fuel consumption is changing at a rate of 0.05 gallons per mile.
(d) When going at 65 miles per hour, the rate of fuel consumption is changing at a rate of 0.05 gallons per hour.

_____ If $f(\omega) = e^3\omega^{100}$ then $f'(\omega)$ is
(a) $e^3\omega^{100}$.
(b) $3e^2\omega^{100}$.
(c) $100e^3\omega^{99}$.
(d) $300e^2\omega^{99}$.
(e) $e^3\omega^{100} + 100e^3\omega^{99}$.
(f) $3e^2\omega^{100} + 100e^3\omega^{99}$.
(g) none of the above.

_____ $\lim_{y\to 0} \dfrac{3\sin(y) - \sin(3y)}{y^3} =$
(a) -7.
(b) 0.
(c) 1.
(d) 4.
(e) does not exist.
(f) none of the above.

5. For this problem we are given the following information for the functions f and g:

$x =$	1	2	3	4
$f(x) =$	3	1	4	2
$f'(x) =$	2	4	1	3
$g(x) =$	1	3	2	4
$g'(x) =$	4	2	3	1

(a) (2 points) If f and g both have inverses then $f^{-1}(g^{-1}(2)) =$

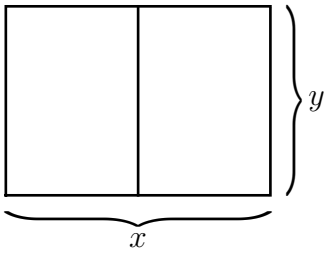
(b) (3 points) If $h(x) = f(x)g(x)$ then $h'(3) =$

(Problem 5 continued)

(c) (3 points) If $k(x) = \dfrac{g(x)}{f(x)}$ then $k'(1) =$

(d) (4 points) If $c(x) = f(g(x))$ then give an equation for the tangent line to $c(x)$ when $x = 4$.

6. (10 points) You want to construct a fenced off area that will be divided down the middle by another stretch of fence (see the picture below). Every foot of fencing used costs 1 dollar. Find the measurements of x and y (in terms of feet) that will maximize the total area enclosed in the fenced off area, given you have a budget of 120 dollars for buying fencing. [Remember you have to buy the fence all the way around the outside as well as for the stretch in the middle.]



7. (10 points) What values of a and b will make the following function *differentiable* at $x = 0$? (Hint: recall that for a function to be differentiable at a point the function has to be continuous at that point and the derivatives from both sides have to agree (i.e., to avoid a sharp bend).)

$$f(x) = \begin{cases} 3\tan(x) - 2\sec(x) & \text{when } x < 0; \\ \\ ae^{2x} + bx & \text{when } x \geq 0. \end{cases}$$

SCRATCH PAPER