Calculus, in itself, is simple. What bogs many people down is algebra and organization. Most of the problems presented below are extremely common, so consider each one, and if it's a mistake you make frequently, then internalize the correct approach.

- Most operations do *not* distribute over addition. $\sqrt{x+y} \neq \sqrt{x} + \sqrt{y}$, $\sin(x+y) \neq \sin x + \sin y$, $\ln(x+y) \neq \ln x + \ln y$, etc. There are only two situations where additive distribution is actually appropriate:
  - $a(b+c) = ab + ac$
  - $\ln(ab) = \ln a + \ln b$ (note this is not an addition on the left side of the equation)
- The transcendental functions (sin, ln, cos, etc.) take an *argument*; they are not "multiplied by" what follows them. $\sin(5x)$ is "sine of $5x$", not "sine times $5x$". You can't manipulate the function in isolation of its argument, for instance, you cannot perform the division $\frac{\sin(5x)}{\sin}$.
  - The solution: to "clear" a function, don't divide by it, but compose it with its inverse function; e.g. above one might take the arcsine to get $\arcsin(\sin(5x)) = 5x$.
- When solving an equation, remember that you must apply the same operation to both sides, and to the entirety of both sides. Below are some examples of improper equation manipulations, as well as a more constructive approach to each:
  - $3x = 6 - y \not\Longrightarrow x = 2 - y$: the division by 3 is not applied consistently on the right side of the equation.
    - $3x = 6 - y \Longrightarrow x = 2 - \frac{y}{3}$.
  - $e^x - y = 30 \not\Longrightarrow x - y = \ln 30$: Logarithm is not applied consistently on the left side of the equation.
    - $e^x - y = 30 \not\Longrightarrow x - \ln y = \ln 30$: Applying the logarithm to the entire left side is *not* the same as applying it to individual elements of the left side (see above, distribution over addition).
    - $e^x - y = 30 \Longrightarrow \ln(e^x - y) = \ln 30$: This is algebraically correct but not terribly useful.
    - $e^x - y = 30 \Longrightarrow e^x = 30 + y \Longrightarrow x = \ln(30 + y)$: By getting e^x alone on one side, we can take the logarithm of both sides and do something productive.
- Do not use equality, or $\frac{d}{dx}$, as "train of thought" connectors. A common occurrance of this is as the phrase, for instance "$f(x) = 3x^2 = \frac{d}{dx} \quad 6x$". This tells a fragmented story which is confusing to all, and fatal in complicated calculations. If performing calculations solely on an algebraic expression without $\frac{d}{dx}$ appearing in it, do those calculations on a line alone (e.g. "$f(x) = 3x^2$" above). Then write, on a different line and as a seperate equation, that you are performing the differentiation (e.g. "$f'(x) = 6x$ or $\frac{d}{dx}3x^2 = 6x$").

- If performing a chain rule decomposition with functions, avoid using the same letter as the argument of f as is the variable in use. For instance, breaking down $\sin\tan x$ as a composition, one might write $g(x) = \tan x$ and $f(u) = \sin u$. This will prevent mistakes of accidentally using $f'(x)$ instead of $f'(g(x))$.

- Infinity is not a number; $f(\infty)$ is not a mathematically significant phrase. Use the limit $\lim_{x\to\infty} f(x)$ wherever you're tempted to do so. The symbol ∞ is only appropriate in the following set-piece idioms:

  - $\lim_{x\to\pm\infty} f(x)$: evaluation of a function's long-term behavior.
  - $\lim_{x\to a} f(x) = \pm\infty$: indicates that the function $f(x)$ grows (or shrinks) without bound as x gets close to a. In particular, despite the presence of the symbol $=$, this is *not* an equality, and $\lim_{x\to a} f(x)$ does not in fact exist.
  - $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\infty}{\infty}$: indicates that both f and g have infinite discontinuities at a; useful as a justification of L'Hospital's rule.

- L'Hospital's rule is *only* applicable when a limit's terms are indeterminate of the form $\frac{\infty}{\infty}$ or $\frac{0}{0}$. Most other limits are directly evaluatable or established not to exist. Some limits which you can not apply L'Hospital's rule to are:

  - $\frac{k}{0}$ for nonzero k; this limit does not exist and represents an infinite discontinuity (possibly of a $+\infty$ type, possibly $-\infty$, possibly neither).
  - $\frac{\infty}{k}$ for finite k; this limit does not exist and represents an infinite discontinuity (possibly of a $+\infty$ type, possibly $-\infty$, possibly neither).
  - $\frac{0}{k}$ for nonzero k; this limit is zero.
  - $\frac{k}{\infty}$ for finite k; this limit is zero.

I examined 85 separate and distinct calculus books. I looked at all of their prefaces, all of their applications of maxima and minima, and all of their treatments of L'Hospital's Rule. By the way, I found five different spellings of L'Hospital. There were the two you would expect [L'Hospital and L'Hôpital], and Lhospital, as L'Hospital sometimes spelled his name. In addition, one author, not wanting to take chances, had it L'Hôspital, and one thought it was Le Hospital.

—Underwood Dudley