There are several problems which you find yourself solving over and over. Here are approaches to each of them.

- Taking a derivative
  - Put $\frac{d}{dx}$ in front of things *to be differentiated*. This will help you keep straight what you've already taken the derivative of, and what you haven't.
  - Before performing any differentiation, check to see if you can simplify. If your differentiand is a quotient, can you cancel on top and bottom? If it's got a sum in it, are terms combinable? The more cleanup you can do in advance, the less messy your actual differentiation will be.
  - Look at the *outermost* operation in the thing being differentiated. Is it a product or a quotient? Then you're looking at a product or quotient rule, no matter what the individual operands are. If it's anything besides a product or quotient, you're probably looking at the chain rule.
  - After applying one of the constructive rules (product, quotient, or chain), look at what still needs to be differentiated (they should have $\frac{d}{dx}$ in front of them if you're practicing proper organization!), and take inventory of what you still need to find out. You may wish to solve these derivatives separately, and then only after solving then in isolation tip them back into your original result.
  - Below is an example of a "good problem-solving discipline". Text between equations is explanatory only; you need not be this verbose. This is the answer to an exam problem:

    Let $w = (\sin t)\arctan(\ln t)$. What is $\frac{dw}{dt}$?
    This is a product-rule problem, since at the outermost level, w is a product (of the expressions $\sin t$ and $\arctan(\ln t)$). Our product-rule expansion decomposes $\frac{dw}{dt}$ into and expression involving two simpler derivatives.

$$\begin{aligned}\frac{dw}{dt} &= \frac{d}{dt}\left(\sin t \arctan(\ln t)\right)\\ &= \left(\frac{d}{dt}\sin t\right)\arctan(\ln t) + (\sin t)\frac{d}{dt}\left(\arctan(\ln t)\right)\end{aligned}$$

    Of these two derivatives, $\frac{d}{dt}\sin t = \cos t$ is easy, but $\frac{d}{dt}\arctan(\ln t)$ requires a chain rule decomposition, which for organization we solve separately. Letting $u = \ln t$, we have $\frac{du}{dt} = \frac{1}{t}$, so we may use the chain rule on $\arctan(\ln t)$:

$$\begin{aligned}\frac{d}{dt}\arctan \ln t &= \frac{d}{dt}\arctan u\\ &= \frac{du}{dt}\frac{d}{du}\arctan u\\ &= \frac{1}{t}\cdot\frac{1}{1+u^2} = \frac{1}{t(1+\ln^2 t)}\end{aligned}$$

Now, we can put this expression back into our original product-rule expansion:

$$\begin{aligned}\frac{dw}{dt} &= \left(\frac{d}{dt}\sin t\right)\arctan(\ln t) + (\sin t)\frac{d}{dt}\left(\arctan(\ln t)\right)\\ &= \cos t \arctan(\ln t) + \sin t \frac{1}{t(1+\ln^2 t)}\end{aligned}$$

  - Advanced time-saving tips: these are two very easy chain-rule results which occur extremely frequently: $\frac{d}{dx}f(x+k) = f'(x+k)$ and $\frac{d}{dx}f(kx) = kf'(kx)$. Thus, for instance, $\frac{d}{dx}\arcsin(3x) = 3 \cdot \frac{1}{\sqrt{1-(3x)^2}}$, or $\frac{d}{dx}\left[(x+3)^{10}\right] = 10(x+3)^9$.

- Sketching a function
  - Start by identifying the function's domain. Square roots of negative numbers and divisions by zero are the most conspicuous offenders.
  - If possible, figure out where the function is positive, negative, zero, and vertically asymptotic. Draw the zeroes on your graph immediately; draw dotted lines to indicate vertical asymptotes. This information alone should give you a pretty good idea where your graph is at every value, but don't draw it in until you have a clearer picture.
  - Determine your graph's long-term positive and negative behavior (to the extent your domain requires it; for instance, $f(x) = \sqrt{x}$ doesn't need its behavior at large negative numbers scoped out. Write down what this behavior is somewhere, but don't draw it into your graph yet.
  - Determine your function's first derivative. Then try to figure out its sign everywhere; identify zeroes of the derivative and non-differentiable points (critical points); the signs of your function near critical points should tell you what sort of critical they are. It may not be a bad idea to draw in maxima and minima; you may not know their exact y-value, but the fact that they're extrema is more important.
  - Armed with this knowledge, you should be able to draw a pretty good graph. You know which half of the plane your graph is in at each x-value and whether it's increasing and decreasing, and you have a lot of dots to connect up (zeroes, asymptotes, extrema). Be sure to do the appropriate strange things at critical points which are not extrema as well. After connecting up all your features, fill in the left and the right sides of the graph with your previously recorded long-term behavior.
  - Advanced topic: the second derivative is not actually all that useful unless you're explicitly asked to note concavity or points of inflection. Generally a sketch drawn with the information gleaned above will simply be forced to have the right concavities.

- Optimization

- Identify the domain of your problem (especially relevant in modeling: what choices make sense?) and, if a modeling problem, explicitly determine the function being optimized, and whether it's a maximum or minimum.
- Differentiate the function, and find the values of x such that $f'(x)$ is zero or undefined. Write these as "possible optima".
- Remove any possible optima not in the domain, and add the endpoints of the domain (including $-\infty$ and ∞ when applicable).
- Evaluate $f(x)$ at each possible optimum (using limits in the case of $-\infty$ and ∞). When $f(x)$ (or a limit) does not exist, investigate whether it does not exist in a way getting very positive or very negative.
- The maximum is the largest $f(x)$ value achieved at your potential optima (or nonexistent if $f(x)$ is getting very positive somewhere); likewise with the minimum for small/negative numbers.