Homework 2  
Math 183, UCSD, Fall 2017  
Due on Friday 13th, October 12:50pm  
Staple pages together  

Exercise 1  
The Behavioral Risk Factor Surveillance System (BRFSS) is an annual telephone survey designed to identify risk factors in the adult population and report emerging health trends. The following table summarizes two variables for the respondents: health status and health coverage, which describes whether each respondent had health insurance.  

<table>
<thead>
<tr>
<th>Health Coverage</th>
<th>Health Status</th>
<th>Excellent</th>
<th>Very good</th>
<th>Good</th>
<th>Fair</th>
<th>Poor</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>No</td>
<td>Excellent</td>
<td>459</td>
<td>727</td>
<td>854</td>
<td>385</td>
<td>99</td>
<td>2524</td>
</tr>
<tr>
<td></td>
<td>Very good</td>
<td>127</td>
<td>542</td>
<td>672</td>
<td>124</td>
<td>43</td>
<td>2048</td>
</tr>
<tr>
<td></td>
<td>Good</td>
<td>54</td>
<td>86</td>
<td>150</td>
<td>74</td>
<td>16</td>
<td>251</td>
</tr>
<tr>
<td></td>
<td>Fair</td>
<td>8</td>
<td>14</td>
<td>34</td>
<td>10</td>
<td>3</td>
<td>75</td>
</tr>
<tr>
<td></td>
<td>Poor</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>Total</td>
<td>Excellent</td>
<td>457</td>
<td>127</td>
<td>54</td>
<td>86</td>
<td>8</td>
<td>675</td>
</tr>
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<td>672</td>
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</tr>
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<td></td>
<td>Fair</td>
<td>8</td>
<td>14</td>
<td>34</td>
<td>10</td>
<td>3</td>
<td>75</td>
</tr>
<tr>
<td></td>
<td>Poor</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>4674</td>
<td>6972</td>
<td>5675</td>
<td>2019</td>
<td>677</td>
<td>20000</td>
</tr>
</tbody>
</table>

1. If we draw one individual at random, what is the probability that the respondent has excellent health and doesn’t have health coverage?

\[
P(\text{excellent and no health coverage}) = \frac{459}{20000} = 0.02295\]

2. If we draw one individual at random, what is the probability that the respondent has excellent health or doesn’t have health coverage?

\[
P(\text{excellent or no health coverage}) = \frac{4657 + 2524 - 459}{20000} = 0.3361\]
Exercise 2

After a statistics course, 86% of students know the formula for standard deviation. Among those who know this formula, 80% passed, but only 56% of those students who did not know the formula passed.

1. Construct a tree diagram of this scenario.

2. Compute the probability that a student know the formula for standard deviation if it is known that he/she passed.

\[
P(\text{know formula for sd} \mid \text{pass}) = \frac{P(\text{know formula and pass})}{P(\text{pass})}
\]

\[
= \frac{P(\text{know formula and pass})}{P(\text{know formula and pass}) + P(\text{don't know formula and pass})}
\]

\[
= \frac{(0.86)(0.8)}{(0.86)(0.8) + (0.14)(0.56)} \approx 0.8977
\]
Exercise 3

Five fair dice are rolled. What is the probability that exactly three faces show one number and two faces show a second number?

There are $5 \times 6$ possibilities for full houses (11122, 11133, 11144 etc..)

Fix a full house type (say aabb), need to determine the number of possible rolls that generate it \{a,a,a,b,b\} \{a,a,b,a,b\} \{a,a,b,b,a\} etc......

To determine this, you can just say it boils down to placing b's in a word of size 5 with letter "a" and "b". Hence, it's 5 choose 2 (equivalent to 5 choose 3).

End up with the probability $P(\text{Full House}) = \frac{6 \cdot (\binom{5}{2})}{6^5}$

= 0.03858
Exercise 4

Imagine you have an urn containing 5 red, 3 blue, and 2 orange marbles in it.

1. What is the probability that the first marble you draw is blue?

\[
P(\text{first marble is blue}) = \frac{\text{# of blue marbles}}{\text{# total marbles}} = \frac{3}{10} \]

2. Suppose you drew a blue marble in the first draw. If drawing with replacement, what is the probability of drawing a blue marble in the second draw?

\[
P(\text{draw blue in the second | I drew blue in the first}) = \frac{3}{10}, \quad \text{since draws with replacement, } \frac{\text{blue & total marbles doesn't change}.}{\text{# of total marbles doesn't change}} \]

3. Suppose you instead drew an orange marble in the first draw. If drawing with replacement, what is the probability of drawing a blue marble in the second draw?

\[
P(\text{blue marble in the second | first was orange}) = \frac{2}{10} \quad \text{same reason as (2)}. \]

4. If drawing with replacement, what is the probability of drawing two blue marbles in a row?

\[
P(\text{2 blue in a row}) = \left( \frac{3}{10} \right) \left( \frac{3}{10} \right) = \frac{9}{100}, \quad \text{since drawing with replacement, second draw exactly the same as first draw} \]

5. When drawing with replacement, are the draws independent? Explain.

They're independent cause what we get on the first draw does not affect what we get on the second.
Exercise 5

Consider a box containing four balls: one red, one green, one blue, and one tricolor (=red, green and blue). You draw one ball from the box. Consider the three events:

\[ R = \{ \text{the drawn ball contains red} \} \]
\[ G = \{ \text{the drawn ball contains green} \} \]
\[ Y = \{ \text{the drawn ball contains red and green} \} \]

1. Are \( R \) and \( G \) independent?

\[ P(R) = \frac{1}{2}, \quad P(R | G) = \frac{1}{3} \]
\[ P(R) = P(R | G), \quad R \text{ and } G \text{ are independent.} \]

2. Are \( G \) and \( Y \) independent?

\[ P(G) = \frac{1}{2}, \quad P(G | Y) = 1 \quad ; \quad P(G) \neq P(G | Y) \]

\( G \) and \( Y \) are not independent

3. Are \( R \) and \( Y \) independent?

\[ P(R) = \frac{1}{2} \]
\[ P(R | Y) = 1, \quad P(R) \neq P(R | Y) \]

\( R \) and \( Y \) are not independent.

4. Let \( A, B, C \) be three events. Is the following statement true? Justify your answer.

"If \( A \) and \( B \) are independent, and \( B \) and \( C \) are independent, then \( A \) and \( C \) are independent."

No, it’s not true.

An counterexample is following: If tossing a dice once, let \( A \) denote getting 1 , \( B = \{ \text{getting an even number} \} \)
\( C = \{ \text{getting an odd number} \} \) . \( A \) and \( B \), \( B \) and \( C \) are independent , but \( A \) and \( C \) are not independent.
Exercise 6

You play the following game from a well-shuffled deck of 52 cards. If you draw a black card, you win $1. If you draw a heart, you win $4. For any diamond, you win $7, plus an additional $15 for the king or ace of diamonds.

1. Create a probability model for the amount you win playing this game. Find the expected value and standard deviation for this model.

<table>
<thead>
<tr>
<th>$X$</th>
<th>1</th>
<th>4</th>
<th>7</th>
<th>7+15=22</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p(X)$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{1}{4}(\frac{13}{13})$</td>
<td>$\frac{1}{4}(\frac{1}{13})$</td>
</tr>
</tbody>
</table>

$E(X) = \sum X \cdot p(X) = \frac{1}{2} + 4(\frac{1}{4}) + 7(\frac{1}{4})(\frac{13}{13}) + 22(\frac{1}{4})(\frac{1}{13}) \approx 3.83$

$sd(X) = \sqrt{Var(X)}, \text{ where } Var(X) = \sum (X - E(X))^2 p(X) = (1-3.83)^2 \cdot \frac{1}{2} + (4-3.83)^2 \cdot \frac{1}{4} + (7-3.83)^2 \cdot \frac{1}{4}(\frac{13}{13}) + (22-3.83)^2 \cdot \frac{1}{4}(\frac{1}{13}) \approx 18.84$

So $sd(X) = \sqrt{18.84} \approx 4.34$

2. On average, what is the most a person should be willing to pay to play this game if the goal is to make a profit?

It should be no more than the expected amount of win, which is $3.83$

3. Assume there is no fee to play. If you play the game each day of the week (7 days/week), what do you expect your weekly earnings to be? What is the standard deviation of the weekly totals?

We are exploring $X_1 + X_2 + \cdots + X_7$, suppose each day of the week are independent and identical.

$E(X_1 + X_2 + \cdots + X_7) = E(X_1) + \cdots + E(X_7) = 7(3.83) = 26.81$

$Var(X_1 + \cdots + X_7) = 7 Var(X_1) = 131.88 \text{ by independence}$

So, $sd(X_1 + \cdots + X_7) = \sqrt{131.88} = 11.48$