Today: Chapter 6 (continued)

- Inference for the difference of two proportions
- Conditions for inferring difference of proportions
- Pooled proportions for hypothesis testing (only!)
Statistics in the Large

Mean

- Sample from 1 population
- Sample from 2 populations

Proportion

- Sample from 1 population
- Sample from 2 populations

Populations not paired

Populations paired
Two-Sample Proportion Inference

Before, we either had 1 population, or two, but we knew the parameter for the second population. Now, we don’t know anything about either population.
Two-Sample Proportion Inference

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Typically, when we compare $p_1$ and $p_2$ (or $\mu_1$ and $\mu_2$), we think about $p_1 - p_2$.
For example, if you care about $p_1 > p_2$, then explore $p_1 - p_2 > 0$. 
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For example, if you care about $p_1 > p_2$, then explore $p_1 - p_2 > 0$.

We might try to infer this using a C.I. for $\hat{p}_1 - \hat{p}_2$, or we might run a hypothesis test with $H_0: p_1 - p_2 = 0$. 
Unpaired Independent Populations

Population 1
Parameters $p_1$
Sample 1 (size $n_1$)
Statistics $\hat{p}_1$

Population 2
Parameters $p_2$
Sample 2 (size $n_2$)
Statistics $\hat{p}_2$

(Note: the samples may have different sizes)

Normal distribution
$E(\hat{p}_1) = p_1$
$SE(\hat{p}_1) = \sqrt{\frac{p_1 q_1}{n_1}}$.

What does the sampling distribution of $\hat{p}_1 - \hat{p}_2$ look like?
Shape?
Center?
Spread?

Normal distribution
$E(\hat{p}_2) = p_2$
$SE(\hat{p}_2) = \sqrt{\frac{p_2 q_2}{n_2}}$. 
Unpaired Independent Populations

Population 1
Parameters $p_1$

Sample 1 (size $n_1$)
Statistics $\hat{p}_1$

Population 2
Parameters $p_2$

Sample 2 (size $n_2$)
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Sample 2 (size $n_2$)
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Unpaired Independent Populations

If $X$ and $Y$ are independent random variables with Normal distributions, then $X - Y$ is also Normal. In addition, $E(X - Y) = E(X) - E(Y)$, and

$$SD(X - Y) = \sqrt{Var(X - Y)} = \sqrt{SD(X)^2 + SD(Y)^2}.$$
Unpaired Independent Populations

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$$SD(X - Y) = \sqrt{Var(X - Y)} = \sqrt{SD(X)^2 + SD(Y)^2}.$$ 

So if $\hat{p}_1 \sim N \left( p_1, \sqrt{\frac{p_1 q_1}{n_1}} \right)$ and $\hat{p}_2 \sim N \left( p_2, \sqrt{\frac{p_2 q_2}{n_2}} \right)$ are independent, we get

$$\hat{p}_1 - \hat{p}_2 \sim N \left( p_1 - p_2, \sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}} \right).$$
Unpaired Independent Populations

Normal distribution

\[ E(\hat{p}_1) = p_1 \]
\[ SE(\hat{p}_1) = \sqrt{\frac{p_1 q_1}{n_1}}. \]

Normal distribution

\[ E(\hat{p}_2) = p_2 \]
\[ SE(\hat{p}_2) = \sqrt{\frac{p_2 q_2}{n_2}}. \]

For unpaired data, the sampling distribution of \( \hat{p}_1 - \hat{p}_2 \) is:

- Normal
- \( E(\hat{p}_1 - \hat{p}_2) = p_1 - p_2 \)
- \( SE(\hat{p}_1 - \hat{p}_2) = \sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}} \)
Drill, Baby, Drill

A 2010 survey asked 827 random voters in California how they feel about drilling for oil off the coast of CA. Of the 438 college graduates in the sample, 154 approved. Of the 389 who didn’t graduate from college, 132 were in favor.

Find a 95% C.I. for the difference in the proportions of college and non-college California grads who support drilling.
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Find a 95% C.I. for the **difference in the proportions** of college and non-college California grads who support drilling.

Let $p_1$ be the proportion of CA college grads that support drilling. Let $p_2$ be the proportion of CA non-college grads that support drilling.
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Find a 95% C.I. for the difference in the proportions of college and non-college California grads who support drilling.

Let \( p_1 \) be the proportion of CA college grads that support drilling. Let \( p_2 \) be the proportion of CA non-college grads that support drilling.

We found \( \hat{p}_1 = \frac{154}{438} \approx 35.16\% \) and \( \hat{p}_2 = \frac{132}{389} \approx 33.93\% \). So

\[ \hat{p}_1 - \hat{p}_2 \approx 1.23\%. \]
Recall that the sampling distribution of the difference $\hat{p}_1 - \hat{p}_2$ is

$$N \left( p_1 - p_2, \sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}} \right).$$
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To build a confidence interval, we will need to estimate the SE since we don’t know $p_1$ or $p_2$. 
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\]

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As usual, we use the point estimate \( SE_{\hat{p}_1 - \hat{p}_2} \approx \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}} \).

As before, we start at our estimate and reach out a certain number of SE’s:

\[
(\hat{p}_1 - \hat{p}_2) \pm z^* \times SE_{\hat{p}_1 - \hat{p}_2}.
\]
We found $\hat{p}_1 \approx 35.16\%$ and $\hat{p}_2 \approx 33.93\%$, so $\hat{p}_1 - \hat{p}_2 \approx 1.23\%$.

We find $SE = \sqrt{\frac{35.16 \times 64.84}{438} + \frac{33.93 \times 66.07}{389}} \approx 3.312\%$. 
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For a 95\% C.I., we must reach \( z^* = 1.96 \) SE’s:

\[
(\hat{p}_1 - \hat{p}_2) \pm z^* \times SE_{\hat{p}_1 - \hat{p}_2} = 1.23 \pm 1.96 \times 3.312 \\
= (-5.26\%, 7.72\%).
\]

```r
> prop.test(x = c(154,132), n=c(438,389), conf.level=0.95, correct=F)

2-sample test for equality of proportions without continuity correction

data:  c(154, 132) out of c(438, 389)
X-squared = 0.13703, df = 1, p-value = 0.7113
alternative hypothesis: two.sided
95 percent confidence interval:
-0.05264371 0.07717682
sample estimates:
prop 1  prop 2
0.3515982  0.3393316
```
But Wait! When is the Sampling Distribution What We Claim?

To get each of the individual sampling distributions to be Normal, in each sample we need:

• Independence (usually shown through Randomization and <10% Conditions)
• At least 10 successes and failures

To use the $Var(X - Y) = Var(X) + Var(Y)$ formula to find $SE_{\hat{p}_1 - \hat{p}_2}$, we need

• Independence between the two samples
Below is given two samples (A and B) and a proportion of interest that you want to compare across the two groups. Which of the following setups will violate the independence required between the two samples?

1. A: Random Californians,
   B: Random Texans;
   percent with college degree in CA vs TX residents

2. A: Random married men,
   B: The wives of those married men;
   percent with college degrees in married men and married women

3. A: Random adults that have kids,
   B: Kids of those adults;
   percent that believe in God in adults vs kids.

4. A: Random people in Canada,
   B: Random people in the U.S.;
   percent that enjoy ice hockey in Canada vs U.S.

5. A: Random people not on antidepressants,
   B: Those same people after taking antidepressants;
   percent of people that are happy off and on antidepressants.

Answer: 2, 3, and 5.
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Answer: 2, 3 and 5.
Your Turn!

Suppose $X$ and $Y$ are independent random variables where $X = \mathcal{N}(4, 3)$ and $Y = \mathcal{N}(2, 1)$.
What will the distribution of $X - Y$ look like?

1. $\mathcal{N}(2, 2)$
2. $\mathcal{N}(2, 4)$
3. $\mathcal{N}(2, \sqrt{10})$
4. $\mathcal{N}(-2, 4)$
5. $\mathcal{N}(-2, -2)$

Answer: 3. $\mathcal{N}(2, \sqrt{10}) = \mathcal{N}(2, \sqrt{10}) = \mathcal{N}(2, \sqrt{10})$
Your Turn!

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1. $N(2, 2)$
2. $N(2, 4)$
3. $N(2, \sqrt{10})$
4. $N(-2, 4)$
5. $N(-2, -2)$

Answer: 3. $N(4 - 2, \sqrt{3^2 + 1^2}) = N(2, \sqrt{10})$
Your Turn!

You create a 90% C.I. for a difference in the proportion of Democrats and Republicans that enjoy the TV personality Stephen Colbert. You find the C.I. for $p_{dem} - p_{rep}$ is (1%, 5%). What is the best way to report this?

1. 90% of the time, Democrats are about 1 to 5% more likely to enjoy S. Colbert.

2. 90% of the time, the percentage difference in those who enjoy S. Colbert (Democrats vs Republicans) will be between 1 and 5%.

3. The difference in the percent of Democrats and Republicans who enjoy S. Colbert is between 1 and 5%.

4. I am 90% confident that the percentage of Democrats who enjoy S. Colbert is 1 to 5% higher than the percentage of Republicans who enjoy Colbert.
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Answer: 4.
Does sexual orientation affect how much people prefer a certain color? In 2001, researchers explored this question with thousands of college students (source). Suppose the 95% C.I. for

\[ p_{\text{LGBT male that likes pink}} - p_{\text{Straight male that likes pink}} \]

was calculated as \((-0.03, 0.04)\). Which of the following statements are true?

1. There is not a statistically significant difference in the percent of college-aged straight males and college-aged LGBT males who like pink.
2. The probability the true parameter difference lies in this interval is 0.95.
3. The 95% C.I. for difference in the other order

\[ p_{\text{Straight male that likes pink}} - p_{\text{LGBT male that likes pink}} \]

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Answer: 1., 3.
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Answer: 1., 3.
Difference in Proportions: Hypothesis Testing

We are usually interested in whether the proportions are different in our two populations. Thus, we set $H_0: \ p_1 - p_2 = 0$ (or equivalently $p_1 = p_2$).
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Common alternative hypotheses are:

$H_A: p_1 - p_2 > 0$

$H_A: p_1 - p_2 \neq 0$

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In one-sample hypothesis testing, we calculate $Z = \frac{\hat{p} - \text{null value}}{SE\hat{p}}$, so you might expect we would do something similar when we have two samples:

$$Z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{SE\hat{p}_1 - \hat{p}_2} = \frac{(\hat{p}_1 - \hat{p}_2) - 0}{SE\hat{p}_1 - \hat{p}_2}.$$
Pooling Our Data

This is almost correct. But notice that \( SE = \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}} \).

This formula acts like we have two different populations going on. But if we assume \( H_0 \), then our populations are really the same (in relation to the idea we are measuring) since \( p_1 = p_2 \).
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Instead of using \( \hat{p}_1 \) and \( \hat{p}_2 \) in this formula, we create a single statistic

\[
\hat{p}_{pooled} = \frac{\text{# Sucesses}_1 + \text{# Sucesses}_2}{n_1 + n_2}.
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\[ \hat{p}_{pooled} = \frac{\# \text{ Sucesses}_1 + \# \text{ Sucesses}_2}{n_1 + n_2}. \]

*Example:* If we had done hypothesis testing for the California drilling example, we would have written

\[ \hat{p}_{pooled} = \frac{154 + 132}{438 + 389} \approx 34.58\%. \]
Pooling Our Data

So, we actually use

\[ Z = \frac{(\hat{p}_1 - \hat{p}_2) - 0}{SE_{pooled}} \], where

\[ SE_{pooled} = \sqrt{\frac{\hat{p}_{pooled} \hat{q}_{pooled}}{n_1} + \frac{\hat{p}_{pooled} \hat{q}_{pooled}}{n_2}} \]
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Why do we pool?
Pooling Our Data

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where 

\[ SE_{pooled} = \sqrt{\frac{\hat{p}_{pooled}\hat{q}_{pooled}}{n_1} + \frac{\hat{p}_{pooled}\hat{q}_{pooled}}{n_2}} \]

Why do we pool?

The simple answer is that when you find the SE, you want to do this with the best info you have available. Usually, this involves just using \( \hat{p}_1 \) and \( \hat{p}_2 \) in place of \( p_1 \) and \( p_2 \). If you are hypothesis testing, you assume momentarily \( p_1 = p_2 \) and get better approximations by using \( \hat{p}_{pooled} \) in place of both \( \hat{p}_1 \) and \( \hat{p}_2 \).
Back to Drilling

Test the claim that CA college grads (Population 1, sample: 153 of 438 supported) are more interested in drilling than CA non-college grads (Population 2, sample: 132 of 389 supported).
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We set $H_0: p_1 - p_2 = 0$ and $H_A: p_1 - p_2 > 0$.

From before, $\hat{p}_1 - \hat{p}_2 = 1.23\%$ and $\hat{p}_{pooled} = 34.58\%$, so that

$$SE_{pooled} = \sqrt{\frac{34.58 \times 65.42}{438} + \frac{34.58 \times 65.42}{389}} \approx 3.31\%.$$
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Our $z$-score is $\frac{1.23 - 0}{3.31} \approx 0.37$

\begin{verbatim}
1 > pnorm(0.37, lower.tail = F)
[1] 0.3556912
\end{verbatim}

Our $p$-value is $p = 0.3557$. Since 0.356 > 0.05, we do not reject the null. It is possible that both populations support drilling equally.
Back to Drilling

Test the claim that CA college grads (Population 1, sample: 153 of 438 supported) are more interested in drilling than CA non-college grads (Population 2, sample: 132 of 389 supported).

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$$SE_{pooled} = \sqrt{\frac{34.58 \times 65.42}{438}} + \frac{34.58 \times 65.42}{389} \simeq 3.31\%.$$

Our z-score is $\frac{1.23 - 0}{3.31} \simeq 0.37$

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A 2012 study from the National Sleep Foundation explored how much sleep various professions get. The above data explore sleep times for the transportation sector.

Do these data suggest that average Americans (control) are less sleep deprived (< 6 hours/night) than train operators? Do a 95% C.I. and hypothesis test.

Let $\hat{p}_T$ be the proportion of train operators that get <6 hours/night, and $\hat{p}_C$ the same idea in the control group.

$\hat{p}_T = \frac{29}{180} \approx 0.161$

$\hat{p}_C = \frac{35}{292} \approx 0.120$

$\hat{p}_T - \hat{p}_C = 0.041$
Sleep Time

<table>
<thead>
<tr>
<th></th>
<th>Control</th>
<th>Pilots</th>
<th>Truck Drivers</th>
<th>Train Operators</th>
<th>Bus/Taxi/Limo Drivers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Less than 6 hours of sleep</td>
<td>35</td>
<td>19</td>
<td>35</td>
<td>29</td>
<td>21</td>
</tr>
<tr>
<td>6 to 8 hours of sleep</td>
<td>193</td>
<td>132</td>
<td>117</td>
<td>119</td>
<td>131</td>
</tr>
<tr>
<td>More than 8 hours</td>
<td>64</td>
<td>51</td>
<td>51</td>
<td>32</td>
<td>58</td>
</tr>
<tr>
<td>Total</td>
<td>292</td>
<td>202</td>
<td>203</td>
<td>180</td>
<td>210</td>
</tr>
</tbody>
</table>

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Do these data suggest that average Americans (control) are less sleep deprived (< 6 hours/night) than train operators? Do a 95% C.I. and hypothesis test.

Let $p_T$ be the proportion of train operators that get <6 hours of sleep/night, and $p_C$ the same idea in the control group.

$$
\hat{p}_T = \frac{29}{180} \approx 0.161, \quad \hat{p}_C = \frac{35}{292} \approx 0.120, \quad so \ \hat{p}_T - \hat{p}_C = 0.041.
$$
Sleep Time

\[ \hat{p}_T = \frac{29}{180} \approx 0.161, \quad \hat{p}_C = \frac{35}{292} \approx 0.120, \quad \text{so } \hat{p}_T - \hat{p}_C = 0.041. \]

Confidence Interval: Do not use a pooled estimate for the C.I’s:

\[
SE \approx \sqrt{\frac{\hat{p}_T \hat{q}_T}{n_T} + \frac{\hat{p}_C \hat{q}_C}{n_C}}
= \sqrt{\frac{0.161 \times 0.839}{180} + \frac{0.12 \times 0.88}{292}} \approx 0.033.
\]

So,

\[ CI = \hat{p}_T - \hat{p}_C \pm z^* \times SE \]
\[ = 0.041 \pm 1.96 \times 0.033 \]
\[ = (-0.024, 0.106). \]
Sleep Time

\[ \hat{p}_T = \frac{29}{180} \approx 0.161, \quad \hat{p}_C = \frac{35}{292} \approx 0.120, \quad \text{so} \quad \hat{p}_T - \hat{p}_C = 0.041. \]

Hypothesis Test: Set \( H_0: \ p_T - p_C = 0 \) and \( p_T - p_C > 0 \).

Under the null, you can (and should!) pool the data and get

\[ \hat{p}_{pooled} = \frac{29 + 35}{180 + 292} \approx 0.135. \]

\[ SE_{pooled} \approx \sqrt{\frac{\hat{p}_{pooled} \hat{q}_{pooled}}{n_T} + \frac{\hat{p}_{pooled} \hat{q}_{pooled}}{n_C}} \]

\[ = \sqrt{\frac{0.135 \times 0.865}{180} + \frac{0.135 \times 0.865}{292}} \approx 0.032. \]
The $z$-score of data is $Z = \frac{(\hat{p}_T - \hat{p}_C) - 0}{SE_{pooled}} \approx \frac{0.041}{0.032} \approx 1.28$.

Our $p$-value is $p = 0.10$. 

$\text{pnorm}(1.28, \text{lower.tail} = \text{F})$

[1] 0.1002726
Sleep Time

The $z$-score of data is 

\[
Z = \frac{(\hat{p}_T - \hat{p}_C) - 0}{SE_{pooled}} \simeq \frac{0.041}{0.032} \simeq 1.28.
\]

Our $p$-value is $p = 0.10$.

Since $p = 0.10 > 0.05$, we do not reject the null.
It appears that average Americans are not less sleep deprived than train operators.