Exercise I

A Gallup Poll released in December 2010 asked 1019 adults living in the Continental U.S. about their belief in the origin of humans. These results, along with results from a more comprehensive poll from 2001 (that we will assume to be exactly accurate), are summarized in the table below.

<table>
<thead>
<tr>
<th></th>
<th>2010</th>
<th>2001</th>
<th>Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>38%</td>
<td>37%</td>
<td></td>
<td>Humans evolved, with God guiding</td>
</tr>
<tr>
<td>16%</td>
<td>12%</td>
<td></td>
<td>Humans evolved, but God had no part in the process</td>
</tr>
<tr>
<td>40%</td>
<td>45%</td>
<td></td>
<td>God created humans in the present form</td>
</tr>
<tr>
<td>6%</td>
<td>6%</td>
<td></td>
<td>Other/No opinion</td>
</tr>
</tbody>
</table>

At the level $\alpha = 5\%$, test whether or not beliefs on the origin of human life changed since from 2001 to 2010.
Exercise II

Recall that if $X \sim P(\lambda)$ is Poisson distributed, $P(X = x) = e^{-\lambda} \frac{\lambda^x}{x!}$ for all integer $x \geq 0$. Furthermore, $\mathbb{E}(X) = \lambda$ and $Var(X) = \lambda$.

1. Given $X_1, \ldots, X_n \sim iid P(\lambda)$, derive the asymptotic distribution of $\sqrt{n}(\bar{X}_n - \lambda)$.

2. If $X \sim P(\lambda)$, give an explicit formula for $h(\lambda) = P(X \geq 1)$.

3. For $0 < \alpha < 1$, build an asymptotic confidence interval of level $1 - \alpha$ for the parameter of interest $h(\lambda) = P(X \geq 1)$.

Exercise III

We run a Gaussian test on a mean of interest $\mu$ in a population. The test is computed using $n$ independent random variables $X_1, \ldots, X_n$ with Normal distribution $N(\mu, 1)$. We let $Z = \frac{\bar{X}_n - \mu_0}{1/\sqrt{n}}$. We want to test the one-sided hypotheses:

$$H_0: \mu = \mu_0, \quad H_1: \mu > \mu_0.$$ 

We let $\mu_1$ denote the true mean if $H_1$ is true. We fix a confidence level $\alpha$ and we compute the critical value $z^*_\alpha$ such that $P(Z > z^*_\alpha | H_0$ is true) = $\alpha$. (for $\alpha = 5\%$, $z^*_\alpha = 1.64$)

1. What is the distribution of $Z$ under $H_0$? And under $H_1$?

2. After defining what the power of a test is (in words), show that the power $1 - \beta$ of this test can be written as

$$1 - \beta = P \left( Y > z^*_\alpha - \frac{\mu_1 - \mu_0}{1/\sqrt{n}} \right),$$

where $Y$ has distribution $N(0, 1)$.

3. Write $\Delta = \mu_1 - \mu_0$ for the difference of means. Study qualitatively the variations (i.e. increasing/decreasing/constant) of the power of the test when:

   (a) $\alpha$ varies;

   (b) $\Delta$ varies;

   (c) $n$ varies.

4. For $\alpha = 5\%$ and $n = 30$, find the smallest difference of means $\Delta$ that we can differentiate in the test, while guaranteeing a power $1 - \beta \geq 80\%$. 