10.2.5

Let \( Y \) denote the distance between the pipeline and the point of impact.

Let \( X_1 \) denote the number of missiles landing within 20 yards to the left of the pipeline.

Let \( X_2 \) denote the number of missiles landing within 20 yards to the right of the pipeline.

Let \( X_3 \) denote the number of missiles which \( |Y| > 20 \).

By symmetry of \( f_Y(y) \),
\[ P = P(Y \in (20, 0)) = \frac{3}{4} = P(Y \in (0, 20)) \]

So \( P_3 = P(\{Y\} > 20) = 1 - P_1 - P_2 = \frac{8}{18} \)

Therefore
\[ P(X_1 = 2, X_2 = 4, X_3 = 0) = \frac{6!}{2!4!1!0!} \cdot \left( \frac{3}{4} \right)^2 \left( \frac{1}{18} \right)^4 \left( \frac{8}{18} \right)^0 = 0.0089 \]

10.2.6

Let \( X_i, i = 1, 2, 3, 4, 5 \), denote the number of outs, singles, doubles, triples, and home runs, respectively.

\[ P(\text{two outs, two singles, one doubles}) = P(X_1 = 2, X_2 = 2, X_3 = 1, X_4 = 0, X_5 = 0) \]

\[ = \frac{3!}{2!1!1!0!1!} \cdot (0.7137)^2 (0.270)^2 (0.010)^1 (0.002)^0 (0.005)^5 \]

\[ = 0.011 \]

10.2.7 10

(a)
\[ P_1 = P(Y \in [0, \frac{1}{4}]) = \int_{0}^{\frac{1}{4}} 3y^2 \, dy = \frac{1}{64} \]

\[ P_2 = P(Y \in [\frac{1}{4}, \frac{1}{2}]) = \int_{\frac{1}{4}}^{\frac{1}{2}} 3y^2 \, dy = \frac{3}{64} \]

\[ P_3 = P(Y \in [\frac{1}{2}, \frac{3}{4}]) = \int_{\frac{1}{2}}^{\frac{3}{4}} 3y^2 \, dy = \frac{9}{64} \]

\[ P_4 = P(Y \in [\frac{3}{4}, 1]) = \int_{\frac{3}{4}}^{1} 3y^2 \, dy = \frac{3}{64} \]

Then
\[ f_{X_1, X_2, X_3, X_4}(3, 2, 15, 25) = P(X_1 = 3, X_2 = 2, X_3 = 15, X_4 = 25) \]

\[ = \frac{50!}{2!1!15!25!} \cdot (\frac{3}{64})^2 \left( \frac{1}{4} \right)^{15} \left( \frac{1}{16} \right)^{25} = \frac{18}{64} \]

\[ = 0.28125 \]
(b) \( X_3 \) is bin \((50, \frac{19}{64})\). Therefore \( \text{Var}(X_3) = nP(1-P) = 50 \cdot \frac{19}{64} \cdot \frac{45}{64} = 10.44 \)

10.2.8
\[
M_{X_1, X_2, X_3}(t_1, t_2, t_3) = E e^{it_1 X_1 + it_2 X_2 + it_3 X_3} = \sum_{k_1+k_2+k_3=n} \frac{n!}{k_1! k_2! k_3!} p_1^{k_1} p_2^{k_2} p_3^{k_3} \quad 3
\]

10.2.9.
\[
M_{X_1, X_2, X_3}(t_1, 0, 0) = (e^{it_1 P_1} + P_2 + P_3)^n = (e^{it_1 P_1} + 1 - P_1)^n \quad 3
\]

But \((e^{it_1 P_1} + P_1)^n\) is the MGF of a Bin \((n, P_1)\).
Therefore \( X_1 \) is binomial with \( n, P_1 \). Similarly, \( X_2 \sim \text{Bin}(n, P_2) \) \( X_3 \sim \text{Bin}(n, P_3) \)

10.2.10
The log likelihood is \( l(P) = \sum_{i=1}^{t} \frac{ki}{n} \log P_i \). We want \( \max_P l(P) \) s.t. \( P_1 + P_2 + \ldots + P_t = 1 \).
Define Lagrange function \( L(P, \lambda) = l(P) - \lambda (P_1 + P_2 + \ldots + P_t - 1) \).
The partial derivative of \( L(P, \lambda) \) is \( \frac{k_i}{P_i} = \lambda \) for all \( i \). Since \( \sum k_i = \lambda \cdot \sum P_i = \lambda \cdot n \), \( \lambda = \frac{n}{\sum P_i} \) for all \( i \).