Homework 6
Math 11, UCSD, Winter 2018
Due on Tuesday, 6th March

Reading: Chapters 18, 19, and 21 (but skip Section 21.4 and the last example in section 21.3), Chapter 20 and Section 21.4.

Note: for hypothesis testing problems, you must show how you calculated your test statistic. You do not need to show work for the step of going from the test statistic to the \( p \)-value, as this step comes from tables or a calculator. If you are using tables rather than a calculator, it is acceptable just to report an interval containing the \( p \)-value. You should always write a concluding sentence in which you explain your conclusion in the context of the problem.

**Exercise 1**

Several factors are involved in the creation of a confidence interval. Among them are the sample size, the level of confidence, and the margin of error. Which statements are true?

1. For a given sample size, reducing the margin of error will mean lower confidence.

2. For a certain confidence level, you can get a smaller margin of error by selecting a bigger sample.

3. For a fixed margin of error, smaller samples will mean lower confidence.

4. For a given confidence level, a sample 9 times as large will make a margin of error one third as big.
Exercise 2

The Pew Research poll described in Exercise 1 found that 49% of a sample of 799 teens admitted to misrepresenting their age online to access websites and online services. (Treat this as a simple random sample.)

1. Find the margin of error for this poll if we want 95% confidence in our estimate of the percent of American teens who have misrepresented their age online.

2. Explain what that margin of error means.

3. If we only need to be 90% confident, will the margin of error be larger or smaller? Explain.

4. Find that margin of error.

5. In general, if all other aspects of the situation remain the same, would smaller samples produce smaller or larger margins of error?
Exercise 3

The mayor of a small city has suggested that the state locate a new prison there, arguing that the construction project and resulting jobs will be good for the local economy. A total of 183 residents show up for a public hearing on the proposal, and a show of hands finds only 31 in favor of the prison project. What can the city council conclude about public support for the mayor’s initiative?

Exercise 4

A 2011 Gallup poll found that 76% of Americans believe that high achieving high school students should be recruited to become teachers. This poll was based on a random sample of 1002 Americans.

1. Find a 90% confidence interval for the proportion of Americans who would agree with this.

2. Interpret your interval in this context.

3. Explain what “90% confidence” means.
Exercise 5

In preparing a report on the economy, we need to estimate the percentage of businesses that plan to hire additional employees in the next 60 days.

1. How many randomly selected employers must we contact in order to create an estimate in which we are 98% confident with a margin of error of 5%?

2. Suppose we want to reduce the margin of error to 3%. What sample size will suffice?
Exercise 6

Write the null and alternative hypotheses you would use to test each situation.

1. In the 1950s, only about 40% of high school graduates went on to college. Has the percentage changed?

2. Twenty percent of cars of a certain model have needed costly transmission work after being driven between 50,000 and 100,000 miles. The manufacturer hopes that a redesign of a transmission component has solved this problem.

3. We field-test a new-flavor soft drink, planning to market it only if we are sure that over 60% of the people like the flavor.
Exercise 7

In the 1980s, it was generally believed that congenital abnormalities affected about 5% of the nation’s children. Some people believe that the increase in the number of chemicals in the environment has led to an increase in the incidence of abnormalities. A recent study examined 384 children and found that 46 of them showed signs of an abnormality. Is this strong evidence that the risk has increased?

1. Write appropriate hypotheses.

2. Check the necessary assumptions and conditions.

3. Perform the mechanics of the test. What is the \( p \)-value?

4. Explain carefully what the \( p \)-value means in context.

5. What’s your conclusion?

6. Do environmental chemicals cause congenital abnormalities?
Exercise 8

The National Center for Education Statistics monitors many aspects of elementary and secondary education nationwide. Their 1996 numbers are often used as a baseline to assess changes. In 1996, 31% of students reported that their mothers had graduated from college. In 2000, responses from 8368 students found that this figure had grown to 32%. Is this evidence of a change in education level among mothers?

1. Write appropriate hypotheses.

2. Check the assumptions and conditions.

3. Perform the test and find the $p$-value.

4. State your conclusion.
Exercise 9

Census data for a certain county show that 19% of the adult residents are Hispanic. Suppose 72 people are called for jury duty and only 9 of them are Hispanic. Does this apparent underrepresentation of Hispanics call into question the fairness of the jury selection system? Explain.
Exercise 10

Which of the following are true? If false, explain briefly.

1. A very low $p$-value provides evidence against the null hypothesis.

2. A high $p$-value is strong evidence in favor of the null hypothesis.

3. A $p$-value above 0.10 shows that the null hypothesis is true.

4. If the null hypothesis is true, you can’t get a $p$-value below 0.01.
Exercise 11

Environmentalists concerned about the impact of high-frequency radio transmissions on birds found that there was no evidence of a higher mortality rate among hatchlings in nests near cell towers. They based this conclusion on a test using $\alpha = 0.05$. Would they have made the same decision at $\alpha = 0.01$? How about $\alpha = 0.10$? Explain.

Exercise 12

Highway safety engineers test new road signs, hoping that increased reflectivity will make them more visible to drivers. Volunteers drive through a test course with several of the new- and old-style signs and rate which kind shows up the best. a) b) c) d) e) f)

1. Is this a one-tailed or a two-tailed test? Why?

2. In this context, what would a Type I error be?
3. In this context, what would a Type II error be?

4. In this context, what is meant by the power of the test?

5. If the hypothesis is tested at the 1% level of significance instead of 5%, how will this affect the power of the test?

6. The engineers hoped to base their decision on the reactions of 50 drivers, but time and budget constraints may force them to cut back to 20. How would this affect the power of the test? Explain.
**Exercise 13**

Based on meteorological data for the past century, a local TV weather forecaster estimates that the region’s average winter snowfall is 23”, with a margin of error of ±2 inches. Assuming he used a 95% confidence interval, how should viewers interpret this news? Comment on each of these statements:

1. During 95 of the past 100 winters, the region got between 21” and 25” of snow.

2. There’s a 95% chance the region will get between 21” and 25” of snow this winter.

3. There will be between 21” and 25” of snow on the ground for 95% of the winter days.

4. Residents can be 95% sure that the area’s average snowfall is between 21” and 25”.

5. Residents can be 95% confident that the average snowfall during the past century was between 21” and 25” per winter.
Exercise 14

Data collected by child development scientists produced this confidence interval for the average age (in weeks) at which babies begin to crawl:

\[ t\text{-Interval for } \mu \]
\[ (95.00\% \text{ Confidence}) : 29.202 < \mu(\text{age}) < 31.844 \]

1. Explain carefully what the software output means.

2. What is the margin of error for this interval?

3. If the researcher had calculated a 90\% confidence interval, would the margin of error be larger or smaller? Explain.

Exercise 15

Hoping to lure more shoppers downtown, a city builds a new public parking garage in the central business district. The city plans to pay for the structure through parking fees. During a two-month period (44 weekdays), daily fees collected averaged $126, with a standard deviation of $15.

1. What assumptions must you make in order to use these statistics for inference?
2. Write a 90% confidence interval for the mean daily income this parking garage will generate.

3. Interpret this confidence interval in context.

4. Explain what “90% confidence” means in this context.

5. The consultant who advised the city on this project predicted that parking revenues would average $130 per day. Based on your confidence interval, do you think the consultant was correct? Why?
Exercise 16

Congress regulates corporate fuel economy and sets an annual gas mileage for cars. A company with a large fleet of cars hopes to meet the 2011 goal of 30.2 mpg or better for their fleet of cars. To see if the goal is being met, they check the gasoline usage for 50 company trips chosen at random, finding a mean of 32.12 mpg and a standard deviation of 4.83 mpg. Is this strong evidence that they have attained their fuel economy goal?

1. Write appropriate hypotheses.

2. Are the necessary assumptions to make inferences satisfied?

3. Find the $p$-value.

4. Explain what the $p$-value means in this context.

5. State an appropriate conclusion.
Exercise 17

Should you generate electricity with your own personal wind turbine? That depends on whether you have enough wind on your site. To produce enough energy, your site should have an annual average wind speed above 8 miles per hour, according to the Wind Energy Association. One candidate site was monitored for a year, with wind speeds recorded every 6 hours. A total of 1114 readings of wind speed averaged 8.019 mph with a standard deviation of 3.813 mph. You’ve been asked to make a statistical report to help the landowner decide whether to place a wind turbine at this site.

1. Discuss the assumptions and conditions for using Student’s t inference methods with these data. Here are some plots that may help you decide whether the methods can be used.

2. What would you tell the landowner about whether this site is suitable for a small wind turbine? Explain.
Exercise 18

During an angiogram, heart problems can be examined via a small tube (a catheter) threaded into the heart from a vein in the patient’s leg. It’s important that the company that manufactures the catheter maintain a diameter of 2.00 mm. (The standard deviation is quite small.) Each day, quality control personnel make several measurements to test $H_0: \mu = 2.00$ against $H_A: \mu \neq 2.00$ at a significance level of $\alpha = 0.05$. If they discover a problem, they will stop the manufacturing process until it is corrected.

1. Is this a one-sided or two-sided test? In the context of the problem, why do you think this is important?

2. Explain in this context what happens if the quality control people commit a Type I error.

3. Explain in this context what happens if the quality control people commit a Type II error.

The catheter company is reviewing its testing procedure.

4. Suppose the significance level is changed to $\alpha = 0.01$. Will the probability of a Type II error increase, decrease, or remain the same?

5. What is meant by the power of the test the company conducts?

6. Suppose the manufacturing process is slipping out of proper adjustment. As the actual mean diameter of the catheters produced gets farther and farther above the desired 2.00 mm, will the power of the quality control test increase, decrease, or remain the same?

7. What could they do to improve the power of the test?