Math 11
Calculus-Based Introductory Probability and Statistics

Eddie Aamari
S.E.W. Assistant Professor

eaamari@ucsd.edu
math.ucsd.edu/~eaamari/
AP&M 5880A

Today:
• Two-Sample Means Inference
Where we stand: We know how to build C.I.’s and run hypothesis tests for one sample means. So we can build a range of plausible values for a single parameter, or compare a single parameter to a known value.
Where we stand: We know how to build C.I.’s and run hypothesis tests for one sample means. So we can build a range of plausible values for a single parameter, or compare a single parameter to a known value.

Today: Extend these ideas to two parameters (two populations)
Statistics in the Large (Reloaded)

Where we stand: We know how to build C.I.’s and run hypothesis tests for one sample means.
So we can build a range of plausible values for a single parameter, or compare a single parameter to a known value.

Today: Extend these ideas to two parameters (two populations)

Nice thing: The approach we use closely follows the transition we made when looking at two proportions (last class).
Where we stand: We know how to build C.I.’s and run hypothesis tests for one sample means. So we can build a range of plausible values for a single parameter, or compare a single parameter to a known value.

Today: Extend these ideas to two parameters (two populations)

Nice thing: The approach we use closely follows the transition we made when looking at two proportions (last class).

Examples of Two-Populations Problems:

- Average SAT score in men VS women at UCSD
- Average height of aliens on planets X and Y
- Average age of husbands and wives
- Average income of children compared to their parents
Statistics in the Large (Reloaded)

Where we stand: We know how to build C.I.’s and run hypothesis tests for one sample means. So we can build a range of plausible values for a single parameter, or compare a single parameter to a known value.

Today: Extend these ideas to two parameters (two populations)

Nice thing: The approach we use closely follows the transition we made when looking at two proportions (last class).

Examples of Two-Populations Problems:

- Average SAT score in men VS women at UCSD
- Average height of aliens on planets X and Y
- Average age of husbands and wives
- Average income of children compared to their parents

Something should sound different about these examples...
Are Your Two Populations Really Independent?

Two extremes:

- Knowing info about members of one population gives no helpful info about members in the other population (Independent samples, 2-sample T-test)
- The members of the two populations have some direct link where each member of one population is paired with a member of the other (Paired samples, 1-sample T-test)
Are Your Two Populations Really Independent?

Two extremes:

- Knowing info about members of one population gives no helpful info about members in the other population (Independent samples, 2-sample T-test)
- The members of the two populations have some direct link where each member of one population is paired with a member of the other (Paired samples, 1-sample T-test)

<table>
<thead>
<tr>
<th>Pre-weight</th>
<th>Post-weight</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>171</td>
<td>168</td>
<td>-3</td>
</tr>
<tr>
<td>203</td>
<td>204</td>
<td>1</td>
</tr>
<tr>
<td>130</td>
<td>135</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Husband Age</th>
<th>Wife Age</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>22</td>
<td>2</td>
</tr>
<tr>
<td>37</td>
<td>40</td>
<td>-3</td>
</tr>
<tr>
<td>81</td>
<td>72</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Are Your Two Populations Really Independent?

Two extremes:

- Knowing info about members of one population gives no helpful info about members in the other population (Independent samples, 2-sample T-test)
- The members of the two populations have some direct link where each member of one population is paired with a member of the other (Paired samples, 1-sample T-test)

<table>
<thead>
<tr>
<th>Pre-weight</th>
<th>Post-weight</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>171</td>
<td>168</td>
<td>-3</td>
</tr>
<tr>
<td>203</td>
<td>204</td>
<td>1</td>
</tr>
<tr>
<td>130</td>
<td>135</td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Husband Age</th>
<th>Wife Age</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>22</td>
<td>2</td>
</tr>
<tr>
<td>37</td>
<td>40</td>
<td>-3</td>
</tr>
<tr>
<td>81</td>
<td>72</td>
<td>8</td>
</tr>
</tbody>
</table>

To analyze paired data, just do analysis on the differences!
C.I. for the Mean Difference of Paired Samples

You decide to research global warming in the U.S. You choose 62 random cities and look up the high temperature on Jan 1st, 1970 and Jan 1st 2017.

Clearly, the data are paired: hot locations will have high readings in both 1970 and 2016. Cold locations will have low readings both times.

You calculate \( d = \text{temp}_{2016} - \text{temp}_{1970} \) for each location, and find the differences \( \bar{d} = 1.1 \)°F with \( s_d = 4.9 \)°F.

Our differences will follow a T-distribution with \( df = 61 \).

We must calculate \( \bar{d} \pm t^* \times \frac{s_d}{\sqrt{n}} \).
C.I. for the Mean Difference of Paired Samples

You decide to research global warming in the U.S. You choose 62 random cities and look up the high temperature on Jan 1st, 1970 and Jan 1st 2017.

Clearly, the data are paired: hot locations will have high readings in both 1970 and 2016. Cold locations will have low readings both times.
C.I. for the Mean Difference of Paired Samples

You decide to research global warming in the U.S. You choose 62 random cities and look up the high temperature on Jan 1st, 1970 and Jan 1st 2017.

Clearly, the data are paired: hot locations will have high readings in both 1970 and 2016. Cold locations will have low readings both times.

You calculate

\[ d = \text{temp}_{2016} - \text{temp}_{1970} \]

for each location, and find the differences \( d \) have \( \bar{d} = 1.1^\circ F \) with \( s_d = 4.9^\circ F \).
C.I. for the Mean Difference of Paired Samples

You decide to research global warming in the U.S. You choose 62 random cities and look up the high temperature on Jan 1st, 1970 and Jan 1st 2017.

Clearly, the data are paired: hot locations will have high readings in both 1970 and 2016. Cold locations will have low readings both times.

You calculate

\[ d = temp_{2016} - temp_{1970} \]

for each location, and find the differences \( d \) have \( \bar{d} = 1.1^\circ F \) with \( s_d = 4.9^\circ F \).

Our differences will follow a T-distribution with \( df = 62 - 1 = 61 \).
You decide to research global warming in the U.S. You choose 62 random cities and look up the high temperature on Jan 1st, 1970 and Jan 1st 2017.

Clearly, the data are paired: hot locations will have high readings in both 1970 and 2016. Cold locations will have low readings both times.

You calculate
\[ d = \text{temp}_{2016} - \text{temp}_{1970} \]
for each location, and find the differences \( d \) have \( \bar{d} = 1.1^\circ F \) with \( s_d = 4.9^\circ F \).

Our differences will follow a T-distribution with \( df = 62 - 1 = 61 \).

We must calculate \( \bar{d} \pm t^*_{61} \times SED_d \).
From the table, $t_{61} \simeq t_{60} \simeq 2.000$.

$SE = \bar{d} \sqrt{n} = 4.9 \sqrt{62} \simeq 0.622$.

Thus, we have $\bar{d} \pm t_{95\%} \times SE \simeq 1.1 \pm 2 \times 0.622 = (-0.144, 2.344)$.

We are 95% confident that temperature rose, on average (at the same location), between $-0.144^\circ F$ and $2.344^\circ F$ in the U.S. between Jan 1st, 1970 and Jan 1st, 2017.
From the table, $t_{61}^* \approx t_{60}^* \approx 2.000$. 

We are 95% confident that temperature rose, on average (at the same location), between $-0.144^\circ F$ and $2.344^\circ F$ in the U.S. between Jan 1st, 1970 and Jan 1st, 2017.
From the table, \( t^{*}_{61} \simeq t^{*}_{60} \simeq 2.000 \).

\[
SE\bar{d} = \frac{s\bar{d}}{\sqrt{n}} = \frac{4.9}{\sqrt{62}} \simeq 0.622.
\]

Thus, we have

\[
\bar{d} \pm t_{df}^{*} \times SE \\
\simeq 1.1 \pm 2 \times 0.622 \\
= (-0.144, 2.344).
\]
From the table, \( t^*_61 \simeq t^*_60 \simeq 2.000 \).

\[
SE\bar{d} = \frac{s\bar{d}}{\sqrt{n}} = \frac{4.9}{\sqrt{62}} \simeq 0.622.
\]

Thus, we have

\[
\bar{d} \pm t^*_{df} \times SE \\
\simeq 1.1 \pm 2 \times 0.622 \\
= (-0.144, 2.344).
\]

We are 95% confident that temperature rose, on average (at the same location), between \(-0.144^\circ F\) and \(2.344^\circ F\) in the U.S. between Jan 1st, 1970 and Jan 1st, 2017.
Wait! What About the Conditions We Must Check?

Since two-sample paired data reduce to a 1-sample T-interval (or T-test) on the differences, we must simply check our usual conditions on the differences (which are the sample undergoing T-testing).

- Independence: The differences must be independent of one another. Since the differences are tied to the same location/person/couple, we just need those paired units to be independent of one another. This is usually checked via the Randomization Condition and the < 10% Condition.

- Nearly Normal Condition: The differences must look nearly normal. As $n$ gets larger, you can weaken this condition.
Wait! What About the Conditions We Must Check?

Since two-sample paired data reduce to a 1-sample T-interval (or T-test) on the differences, we must simply check our usual conditions on the differences (which are the sample undergoing T-testing).

- Independence: The differences must be independent of one another. Since the differences are tied to the same location/person/couple, we just need those paired units to be independent of one another. This is usually checked via the Randomization Condition and the < 10% Condition.

- Nearly Normal Condition: The differences must look nearly normal. As \( n \) gets larger, you can weaken this condition.

How paired units might fail to be independent of one another:
Choosing married couples that go to the same church, doing before/after experiments on college students, picking cities for the 1970/2016 temperature study that all fall in the same latitude, etc.
Researchers collected IQ data on parents of 36 children identified as “gifted”. Below are the results and histogram of the IQ differences of the parents.

Run a test to see if mothers and fathers of gifted children have different average IQ’s.
IQ of Parents of Gifted Children

The parents were chosen randomly, so the differences will be independent. The histogram appears nearly normal (slight left skew, but $n = 36 > 30$).

If we assume $H_0: \mu_d = 0$, then the average sample differences follow a $t_{35}$ distribution with center 0 and $SE = s_d \sqrt{\frac{1}{n}} = 7.5 \sqrt{\frac{1}{36}} \approx 1.25$.

The $t$-score for our observed difference is $T = \frac{\bar{d} - 0}{SE} = 3.4\frac{1}{1.25} \approx 2.72$.

<table>
<thead>
<tr>
<th></th>
<th>Mother</th>
<th>Father</th>
<th>Diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>118.2</td>
<td>114.8</td>
<td>3.4</td>
</tr>
<tr>
<td>SD</td>
<td>6.5</td>
<td>3.5</td>
<td>7.5</td>
</tr>
<tr>
<td>n</td>
<td>36</td>
<td>36</td>
<td>36</td>
</tr>
</tbody>
</table>
The parents were chosen randomly, so the differences will be independent. The histogram appears nearly normal (slight left skew, but \( n = 36 > 30 \)).
IQ of Parents of Gifted Children

The parents were chosen randomly, so the differences will be independent. The histogram appears nearly normal (slight left skew, but \( n = 36 > 30 \)).

If we assume \( H_0: \mu_d = 0 \), then the average sample differences follow a \( t_{36-1} = t_{35} \) distribution with center 0 and \( SE = \frac{s_d}{\sqrt{n}} = \frac{7.5}{\sqrt{36}} \approx 1.25 \).
IQ of Parents of Gifted Children

The parents were chosen randomly, so the differences will be independent. The histogram appears nearly normal (slight left skew, but \( n = 36 > 30 \)).

If we assume \( H_0: \mu_d = 0 \), then the average sample differences follow a \( t_{36-1} = t_{35} \) distribution with center 0 and \( SE = \frac{s_d}{\sqrt{n}} = \frac{7.5}{\sqrt{36}} \approx 1.25 \).

The \( t \)-score for our observed difference is

\[
T = \frac{\bar{d} - 0}{SE} = \frac{3.4}{1.25} \approx 2.72.
\]
If the alternative hypothesis is \( H_A: \mu_d \neq 0 \), we find the following shaded area.

We get a \( p \)-value \( = 2 \times 0.005048 = 0.0096\% \).

We reject the null hypothesis. It does appear that there is a difference in the average IQ’s of parents of gifted children.
IQ of Parents of Gifted Children

If the alternative hypothesis is \( H_A: \mu_d \neq 0 \), we find the following shaded area.

We get a \( p \)-value \( = 2 \times 0.005048 = 1.0096\% \).
IQ of Parents of Gifted Children

If the alternative hypothesis is \(H_A: \mu_d \neq 0\), we find the following shaded area.

We get a \(p\)-value \(= 2 \times 0.005048 = 1.0096\%\).

We reject the null hypothesis. It does appear that there is a difference in the average IQ’s of parents of gifted children.
Remember: Use Minitab to check, not to generate your answers.

Paired t for the Mean

- Sample size: 36
- Sample mean: 3.4
- Standard deviation: 7.5

Paired t: Options

Paired T-Test and CI

<table>
<thead>
<tr>
<th>Difference</th>
<th>N</th>
<th>Mean</th>
<th>StDev</th>
<th>SE Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>36</td>
<td>3.4</td>
<td>7.50</td>
<td>1.25</td>
</tr>
</tbody>
</table>

95% CI for mean difference: (0.86, 5.94)

T-Test of mean difference = 0 (vs ≠ 0): T-Value = 2.72  P-Value = 0.010
Remember: Use Minitab to check, not to generate your answers.
Unpaired Independent Populations

T-distribution with \( df = n_1 - 1 \) centered at \( \mu_1 \) with \( SE = s_1 \sqrt{n_1} \).

What does the sampling distribution of \( \bar{x}_1 - \bar{x}_2 \) look like?

Shape?
Center?
Spread?

T-distribution with \( df = n_2 - 1 \) centered at \( \mu_2 \) with \( SE = s_2 \sqrt{n_2} \).
Unpaired Independent Populations

(Note: the samples may have different sizes)
Unpaired Independent Populations

Population 1
Parameters: $\mu_1, \sigma_1$

Sample (size $n_1$)
Statistics: $\bar{x}_1, s_1$

Population 2
Parameters: $\mu_2, \sigma_2$

Sample (size $n_2$)
Statistics: $\bar{x}_2, s_2$

(Note: the samples may have different sizes)

What does the sampling distribution of $\bar{x}_1 - \bar{x}_2$ look like?
- Shape?
- Center?
- Spread?

T-distribution with $df = n_1 - 1$
centered at $\mu_1$
with $SE = \frac{s_1}{\sqrt{n_1}}$.

T-distribution with $df = n_2 - 1$
centered at $\mu_2$
with $SE = \frac{s_2}{\sqrt{n_2}}$. 
Amazing Fact

If $\bar{X} = t_{n_1-1}$ and $\bar{Y} = t_{n_2-1}$ are independent random variables both modelled by T-distributions, then $\bar{X} - \bar{Y}$ is also a T-distribution with

$$df = \min(n_1 - 1, n_2 - 1).$$
Amazing Fact

If $\bar{X} = t_{n_1-1}$ and $\bar{Y} = t_{n_2-1}$ are independent random variables both modelled by T-distributions, then $\bar{X} - \bar{Y}$ is also a T-distribution with

$$df = \min(n_1 - 1, n_2 - 1).$$

Furthermore, $\bar{X} - \bar{Y}$ is centered at

$$E(\bar{X} - \bar{Y}) = \mu_1 - \mu_2,$$

and has a SE which is found using the formula for the variance of a difference:

$$SE_{\bar{X} - \bar{Y}} = \sqrt{Var(\bar{X} - \bar{Y})} = \sqrt{Var(\bar{X}) + Var(\bar{Y})} \approx \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}.$$
All C.I.’s Are Variations of One Another

<table>
<thead>
<tr>
<th>C.I. for</th>
<th>Formula</th>
<th>SE</th>
<th>df</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 sample</td>
<td>$\bar{x} \pm t_{df}^* SE\bar{x}$</td>
<td>$\frac{s}{\sqrt{n}}$</td>
<td>$n - 1$</td>
</tr>
<tr>
<td>2 paired samples</td>
<td>$\bar{d} \pm t_{df}^* SE\bar{d}$</td>
<td>$\frac{s}{\sqrt{n}}$</td>
<td>$n - 1$</td>
</tr>
<tr>
<td>2 independent samples</td>
<td>$\bar{x}_1 - \bar{x}<em>2 \pm t</em>{df}^* SE\bar{x}_1 - \bar{x}_2$</td>
<td>$\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$</td>
<td>$\min(n_1 - 1, n_2 - 1)$</td>
</tr>
</tbody>
</table>

Keep in mind: When you change the scenario being discussed, you change the sampling distribution, and hence, the critical value and standard error.

To make C.I.’s of new ideas, we just need to know what the sampling distribution is and we are all set!
All C.I.’s Are Variations of One Another

<table>
<thead>
<tr>
<th>C.I. for</th>
<th>Formula</th>
<th>SE</th>
<th>df</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 sample</td>
<td>$\bar{x} \pm t_{df}^* SE\bar{x}$</td>
<td>$\frac{s}{\sqrt{n}}$</td>
<td>$n - 1$</td>
</tr>
<tr>
<td>2 paired samples</td>
<td>$\bar{d} \pm t_{df}^* SE\bar{d}$</td>
<td>$\frac{s}{\sqrt{n}}$</td>
<td>$n - 1$</td>
</tr>
<tr>
<td>2 independent samples</td>
<td>$\bar{x}_1 - \bar{x}<em>2 \pm t</em>{df}^* SE\bar{x}_1 - \bar{x}_2$</td>
<td>$\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$</td>
<td>$\min(n_1 - 1, n_2 - 1)$</td>
</tr>
</tbody>
</table>

**Keep in mind:** When you change the scenario being discussed, you change the sampling distribution, and hence, the critical value and standard error.
To make C.I.’s of new ideas, we just need to know what the sampling distribution is and we are all set!
But Wait! Any Conditions For Us To Do Inference?

To get each of the individual sampling distributions to be a t-distribution we need (in each sample):

- Independence of items in the sample (usually shown through the randomization and $< 10$
- Nearly normal population distribution (check via sample histogram where larger allows for skew)

To be able to subtract the t approximations for each sampling distribution and use our variance formula to get the SE of the difference:

- Independence of the two samples (no datum in one sample should help you predict any datum in the other sample)
But Wait! Any Conditions For Us To Do Inference?

To get each of the individual sampling distributions to be a t-distribution we need (in each sample):

- Independence of items in the sample (usually shown through the randomization and < 10
- Nearly normal population distribution (check via sample histogram where larger allows for skew)

To be able to subtract the $t$ approximations for each sampling distribution and use our variance formula to get the SE of the difference:

- Independence of the two samples (no datum in one sample should help you predict any datum in the other sample)
Beetle Study (Again!)

We study beetle biodiversity in a pasture. For this, we collect a biodiversity index (Steinhaus index) in 2 different types of parcels:

1. in $n_1 = 12$ parcels where no animal grazes
2. in $n_2 = 13$ parcels with sheeps are grazing

Your get the following data:

\[
\bar{x}_1 = 0.2505 \text{ and } s_1 = 0.0959 \\
\bar{x}_2 = 0.4942 \text{ and } s_2 = 0.1067
\]
We study beetle biodiversity in a pasture. For this, we collect a biodiversity index (Steinhaus index) in 2 different types of parcels:

1. in $n_1 = 12$ parcels where no animal grazes
2. in $n_2 = 13$ parcels with sheeps are grazing

Your get the following data:

$$\bar{x}_1 = 0.2505 \text{ and } s_1 = 0.0959$$

$$\bar{x}_2 = 0.4942 \text{ and } s_2 = 0.1067$$

Build a test with level of confidence $\alpha = 5\%$ to determine if animal grazing influences the biodiversity of beetles.
Beetle Study

1) We build hypotheses for the situation

\[ H_0: \mu_{\text{grazed}} = \mu_{\text{not grazed}} \]

\[ H_A: \mu_{\text{grazed}} \neq \mu_{\text{not grazed}} \]

2) We want to build a hypothesis test using the T-distribution, so we have to check the normality of our population distributions.
Beetle Study

1) We build hypotheses for the situation

\[ H_0: \mu_{\text{grazed}} = \mu_{\text{not grazed}}, \]
\[ H_A: \mu_{\text{grazed}} \neq \mu_{\text{not grazed}}. \]
Beetle Study

1) We build hypotheses for the situation

\[ H_0: \mu_{\text{grazed}} = \mu_{\text{not grazed}}, \]
\[ H_A: \mu_{\text{grazed}} \neq \mu_{\text{not grazed}}. \]

2) We want to build a hypothesis test using the T-distribution, so we have to check the normality of our population distributions.
Beetle Study

1) We build hypotheses for the situation

\[ H_0: \mu_{\text{grazed}} = \mu_{\text{not grazed}}, \]
\[ H_A: \mu_{\text{grazed}} \neq \mu_{\text{not grazed}}. \]

2) We want to build a hypothesis test using the T-distribution, so we have to check the normality of our population distributions.

\[
\begin{array}{c|c|c|c|c}
\text{Steinhaus Index} & 0.1 & 0.2 & 0.3 & 0.4 & 0.5 \\
\text{Frequency} & 0 & 2 & 4 & 0 & 0 \\
\end{array}
\]

\[
\begin{array}{c|c|c|c|c}
\text{Indice de Steinhaus} & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 \\
\text{Fréquence} & 0 & 2 & 4 & 0 & 0 \\
\end{array}
\]
3) T-score your data

\[ T = \frac{\text{point estimate} - \text{null value}}{SE} \]

\[ = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \]

\[ = \frac{0.251 - 0.494}{\sqrt{\frac{0.095^2}{12} + \frac{0.107^2}{13}}} \approx -6.011. \]
3) T-score your data

\[ T = \frac{\text{point estimate} - \text{null value}}{SE} \]

\[ = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \]

\[ = \frac{0.251 - 0.494}{\sqrt{\frac{0.095^2}{12} + \frac{0.107^2}{13}}} \approx -6.011. \]

4) Compute the p-value. Here, \( df = \min(12 - 1, 13 - 1) = 11. \)
3) T-score your data

\[
T = \frac{\text{point estimate} - \text{null value}}{SE} = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{0.251 - 0.494}{\sqrt{\frac{0.095^2}{12} + \frac{0.107^2}{13}}} \approx -6.011.
\]

4) Compute the \( p \)-value. Here, \( df = \min(12 - 1, 13 - 1) = 11 \).

From Minitab, we get a \( p \)-value of order \( 10^{-5} \).
3) T-score your data

\[ T = \frac{\text{point estimate} - \text{null value}}{SE} \]

\[ = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \]

\[ = \frac{0.251 - 0.494}{\sqrt{\frac{0.095^2}{12} + \frac{0.107^2}{13}}} \approx -6.011. \]

4) Compute the \( p \)-value. Here, \( df = \min(12 - 1, 13 - 1) = 11. \)

From Minitab, we get a \( p \)-value of order \( 10^{-5} \).

Since \( p \approx 9 \cdot 10^{-5} \ll 0.05 \), we reject \( H_0 \) and favor \( H_A \).

There is (a very) strong evidence that the animal grazing influences beetle biodiversity.
# A Big Helpful Chart

<table>
<thead>
<tr>
<th>Things to Remember!</th>
<th>Difference in Proportions: Confidence Interval</th>
<th>Difference in Proportions: Hypothesis Test</th>
<th>Difference in Means: Confidence Interval</th>
<th>Difference in Means: Hypothesis Test</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Independent samples (Chapter 22)</strong></td>
<td>Sampling distribution of the differences is Normal! [ SE = \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}} ]</td>
<td>To have the best possible SE, we pool the data! [ z = \frac{(\hat{p}<em>1 - \hat{p}<em>2) - (0)}{SE</em>{pooled}} ] where ( SE</em>{pooled} ) and ( \hat{p}_{pooled} ) were defined last class.</td>
<td>Sampling distribution of the differences is a ( t )-distribution with ( df = \min(n_1 - 1, n_2 - 1) ) [ SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} ]</td>
<td>( t_{df} = ) [ \frac{(\bar{x}_1 - \bar{x}_2) - 0}{SE} ] where ( df = \min(n_1 - 1, n_2 - 1) ) ( SE ) [ SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} ] No pooling is necessary.</td>
</tr>
<tr>
<td><strong>Dependent samples that are paired (Chapter 23)</strong></td>
<td>Not covered in Math 11</td>
<td>Not covered in Math 11</td>
<td>Do a 1-sample ( t )-interval on the differences, ( d ). [ \bar{d} \pm t_{df}^* \cdot \frac{s_d}{\sqrt{n}} ] Check that the differences meet the 2 conditions.</td>
<td>Do a 1-sample ( t )-test on the differences, ( d ). [ t_{df} = \frac{d - 0}{SE} ] Check that the differences meet the 2 conditions.</td>
</tr>
</tbody>
</table>
A Few More Worked Examples

On any inference problem about means, you must do the following:

- Decide on the global setup (1 sample, 2 paired samples, 2 independent samples)
- Decide between CI or hypothesis test (one or two-sided)
- Check if conditions for inference are met
- Determine what sampling distribution we are on ($df = ?$)
- Find the $SE$ and use it in the CI formula or to get the $T$-score
Technology and Food

Researchers wanted to see if using technology while eating would cause people to eat more food, perhaps because they were distracted. 44 patients were divided into equal treatment and control groups. The treatment group played computer Solitaire while eating; the control did not.

The weight (in grams) of food consumed were:

- treatment: mean 52.1, sd 45.1
- control: mean 27.1, sd 26.4

Run a hypothesis test on these data.

Parameters:

Let $\mu_T$ be the average weight of food of people eating while playing Solitaire (if the experiment were repeated for ever). $\mu_C$ is for control.

Hypotheses:

$H_0$: $\mu_T - \mu_C = 0$
$H_A$: $\mu_T - \mu_C > 0$

(On this problem, it is tough to check the Nearly Normal condition in each sample. We likely meet the other conditions. We proceed cautiously.)
Technology and Food

Researchers wanted to see if using technology while eating would cause people to eat more food, perhaps because they were distracted. 44 patients were divided into equal treatment and control groups. The treatment group played computer Solitaire while eating; the control did not.

The weight (in grams) of food consumed were:

- treatment: mean 52.1, sd 45.1
- control: mean 27.1, sd 26.4

Run a hypothesis test on these data.
Technology and Food

Researchers wanted to see if using technology while eating would cause people to eat more food, perhaps because they were distracted. 44 patients were divided into equal treatment and control groups. The treatment group played computer Solitaire while eating; the control did not.

The weight (in grams) of food consumed were:
- treatment: mean 52.1, sd 45.1
- control: mean 27.1, sd 26.4

Run a hypothesis test on these data.

Parameters: Let $\mu_T$ be the average weight of food of people eating while playing Solitaire (if the experiment were repeated for ever). $\mu_C$ is for control.
Technology and Food

Researchers wanted to see if using technology while eating would cause people to eat more food, perhaps because they were distracted. 44 patients were divided into equal treatment and control groups. The treatment group played computer Solitaire while eating; the control did not.

The weight (in grams) of food consumed were:
• treatment: mean 52.1, sd 45.1
• control: mean 27.1, sd 26.4

Run a hypothesis test on these data.

Parameters: Let $\mu_T$ be the average weight of food of people eating while playing Solitaire (if the experiment were repeated for ever). $\mu_C$ is for control.

Hypotheses: $H_0 : \mu_T - \mu_C = 0$ and $H_A : \mu_T - \mu_C > 0$

(On this problem, it is tough to check the Nearly Normal condition in each sample. We likely meet the other conditions. We proceed cautiously.)
Sampling Distribution and Picture: For two means, our curve is a T-distribution with:

\[ df = \min(22 - 1, 22 - 1) = 21, \text{ and } SE = \sqrt{\frac{45.1^2}{22} + \frac{26.4^2}{22}} \approx 11.14. \]
Sampling Distribution and Picture: For two means, our curve is a T-distribution with:

\[ df = \min(22 - 1, 22 - 1) = 21, \quad \text{and} \quad SE = \sqrt{\frac{45.1^2}{22} + \frac{26.4^2}{22}} \approx 11.14. \]

\[
T = \frac{(\bar{x}_T - \bar{x}_C) - 0}{SE} = \frac{52.1 - 27.1}{11.14} = 2.244.
\]

Can you find the P-value using a T-table?!
Sampling Distribution and Picture: For two means, our curve is a T-distribution with:

\[ df = \min(22 - 1, 22 - 1) = 21, \text{ and } SE = \sqrt{\frac{45.1^2}{22} + \frac{26.4^2}{22}} \approx 11.14. \]

\[
T = \frac{(\bar{x}_T - \bar{x}_C) - 0}{SE} = \frac{52.1 - 27.1}{11.14} = 2.244.
\]

Can you find the P-value using a T-table?!

Since 0.018 < 0.05, reject \( H_0 \) in favor of \( H_A \).
It does appear that distracted eating (via technology) leads to greater consumption.
Why Those Warning Labels on Cigarettes?

Researchers were interested if smoking was linked with lower birth weights of babies. They sampled 150 random North Carolina mothers and found the below data.

<table>
<thead>
<tr>
<th></th>
<th>smoker</th>
<th>non-smoker</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean weight (lbs)</td>
<td>6.78</td>
<td>7.18</td>
</tr>
<tr>
<td>st. dev.</td>
<td>1.43</td>
<td>1.60</td>
</tr>
<tr>
<td>sample size</td>
<td>50</td>
<td>100</td>
</tr>
</tbody>
</table>

Find a 90% confidence interval for $\mu_{\text{non-smoke}} - \mu_{\text{smoke}}$. 

$\bar{x}_1 - \bar{x}_2 \pm t^\ast \frac{s}{\sqrt{n_1} + \frac{s_2}{\sqrt{n_2}}}$

Here, $SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \approx 0.258$. 

The sampling distribution for the difference in the sample means is a $T$-distribution with $df = \min(50 - 1, 100 - 1) = 49$. 

\begin{align*}
\bar{x} & \pm t^\ast \frac{s}{\sqrt{n}} \\
6.78 & \pm t^\ast \frac{1.43}{\sqrt{50}} \\
7.18 & \pm t^\ast \frac{1.60}{\sqrt{100}} \\
\end{align*}
Why Those Warning Labels on Cigarettes?

Researchers were interested if smoking was linked with lower birth weights of babies. They sampled 150 random North Carolina mothers and found the below data.

<table>
<thead>
<tr>
<th></th>
<th>smoker</th>
<th>non-smoker</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean weight (lbs)</td>
<td>6.78</td>
<td>7.18</td>
</tr>
<tr>
<td>st. dev.</td>
<td>1.43</td>
<td>1.60</td>
</tr>
<tr>
<td>sample size</td>
<td>50</td>
<td>100</td>
</tr>
</tbody>
</table>

Find a 90% confidence interval for $\mu_{\text{non-smoker}} - \mu_{\text{smoker}}$.

We must find $(\bar{x}_1 - \bar{x}_2) \pm t_{df} \times SE_{\bar{x}_1 - \bar{x}_2}$.
Why Those Warning Labels on Cigarettes?

Researchers were interested if smoking was linked with lower birth weights of babies. They sampled 150 random North Carolina mothers and found the below data.

<table>
<thead>
<tr>
<th></th>
<th>smoker</th>
<th>non-smoker</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean weight (lbs)</td>
<td>6.78</td>
<td>7.18</td>
</tr>
<tr>
<td>st. dev.</td>
<td>1.43</td>
<td>1.60</td>
</tr>
<tr>
<td>sample size</td>
<td>50</td>
<td>100</td>
</tr>
</tbody>
</table>

Find a 90% confidence interval for $\mu_{non-smoke} - \mu_{smoke}$.

We must find $(\bar{x}_1 - \bar{x}_2) \pm t_{df}^* \times SE_{\bar{x}_1 - \bar{x}_2}$.

Here, $SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{1.6^2}{100} + \frac{1.43^2}{50}} \approx 0.258$. 
Researchers were interested if smoking was linked with lower birth weights of babies. They sampled 150 random North Carolina mothers and found the below data.

<table>
<thead>
<tr>
<th></th>
<th>smoker</th>
<th>non-smoker</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean weight (lbs)</td>
<td>6.78</td>
<td>7.18</td>
</tr>
<tr>
<td>st. dev.</td>
<td>1.43</td>
<td>1.60</td>
</tr>
<tr>
<td>sample size</td>
<td>50</td>
<td>100</td>
</tr>
</tbody>
</table>

Find a 90% confidence interval for \( \mu_{\text{non-smoke}} - \mu_{\text{smoke}} \).

We must find \( (\bar{x}_1 - \bar{x}_2) \pm t^*_{df} \times SE_{\bar{x}_1 - \bar{x}_2} \).

Here, \( SE = \sqrt{\frac{s^2_1}{n_1} + \frac{s^2_2}{n_2}} = \sqrt{\frac{1.6^2}{100} + \frac{1.43^2}{50}} \approx 0.258 \).

The sampling distribution for the difference in the sample means is a T-distribution with \( df = \min(50 - 1, 100 - 1) = 49 \).
Need to find the critical value $t_{df}^*$. 

Since $\bar{x}_1 - \bar{x}_2 = 7.18 - 6.78 = 0.4$, we have $CI = 0.4 \pm 1.68 \times 0.258 = (-0.03, 0.83)$.

We are 90% confident that babies born to non-smoking NC women are about 0.83 to -0.03 lbs heavier than babies born to smoking NC women.
Need to find the critical value $t_{df}^*$.

\begin{table}
\centering
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
one tail & 0.100 & 0.050 & 0.025 & 0.010 & 0.005 \\
\hline
two tails & 0.200 & 0.100 & 0.050 & 0.020 & 0.010 \\
\hline
df & & & & & \\
31 & 1.31 & 1.70 & 2.04 & 2.45 & 2.74 \\
32 & 1.31 & 1.69 & 2.04 & 2.45 & 2.74 \\
33 & 1.31 & 1.69 & 2.03 & 2.44 & 2.73 \\
34 & 1.31 & 1.69 & 2.03 & 2.44 & 2.73 \\
35 & 1.31 & 1.69 & 2.03 & 2.44 & 2.72 \\
36 & 1.31 & 1.69 & 2.03 & 2.43 & 2.72 \\
37 & 1.30 & 1.69 & 2.03 & 2.43 & 2.72 \\
38 & 1.30 & 1.69 & 2.02 & 2.43 & 2.71 \\
39 & 1.30 & 1.68 & 2.02 & 2.43 & 2.71 \\
40 & 1.30 & 1.68 & 2.02 & 2.42 & 2.70 \\
41 & 1.30 & 1.68 & 2.02 & 2.42 & 2.70 \\
42 & 1.30 & 1.68 & 2.02 & 2.42 & 2.70 \\
43 & 1.30 & 1.68 & 2.02 & 2.42 & 2.70 \\
44 & 1.30 & 1.68 & 2.02 & 2.41 & 2.69 \\
45 & 1.30 & 1.68 & 2.01 & 2.41 & 2.69 \\
46 & 1.30 & 1.68 & 2.01 & 2.41 & 2.69 \\
47 & 1.30 & 1.68 & 2.01 & 2.41 & 2.68 \\
48 & 1.30 & 1.68 & 2.01 & 2.41 & 2.68 \\
49 & 1.30 & 1.68 & 2.01 & 2.40 & 2.68 \\
50 & 1.30 & 1.68 & 2.01 & 2.40 & 2.68 \\
\hline
\end{tabular}
\end{table}

We find $t_{49}^* = 1.68$.

Since $\bar{x}_1 - \bar{x}_2 = 7.18 - 6.78 = 0.4$, we have $CI = 0.4 \pm 1.68 \times 0.258 = (-0.03, 0.83)$.

We are 90% confident that babies born to non-smoking NC women are about 0.83 to -0.03 lbs heavier than babies born to smoking NC women.
Need to find the critical value $t_{df}^*$. 

<table>
<thead>
<tr>
<th>df</th>
<th>1-tail</th>
<th>0.05</th>
<th>0.025</th>
<th>0.010</th>
<th>0.005</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>1.31</td>
<td>1.70</td>
<td>2.04</td>
<td>2.45</td>
<td>2.74</td>
</tr>
<tr>
<td>32</td>
<td>1.31</td>
<td>1.69</td>
<td>2.04</td>
<td>2.45</td>
<td>2.74</td>
</tr>
<tr>
<td>33</td>
<td>1.31</td>
<td>1.69</td>
<td>2.03</td>
<td>2.44</td>
<td>2.73</td>
</tr>
<tr>
<td>34</td>
<td>1.31</td>
<td>1.69</td>
<td>2.03</td>
<td>2.44</td>
<td>2.73</td>
</tr>
<tr>
<td>35</td>
<td>1.31</td>
<td>1.69</td>
<td>2.03</td>
<td>2.44</td>
<td>2.72</td>
</tr>
<tr>
<td>36</td>
<td>1.31</td>
<td>1.69</td>
<td>2.03</td>
<td>2.43</td>
<td>2.72</td>
</tr>
<tr>
<td>37</td>
<td>1.30</td>
<td>1.69</td>
<td>2.03</td>
<td>2.43</td>
<td>2.72</td>
</tr>
<tr>
<td>38</td>
<td>1.30</td>
<td>1.69</td>
<td>2.02</td>
<td>2.43</td>
<td>2.71</td>
</tr>
<tr>
<td>39</td>
<td>1.30</td>
<td>1.68</td>
<td>2.02</td>
<td>2.43</td>
<td>2.71</td>
</tr>
<tr>
<td>40</td>
<td>1.30</td>
<td>1.68</td>
<td>2.02</td>
<td>2.42</td>
<td>2.70</td>
</tr>
<tr>
<td>41</td>
<td>1.30</td>
<td>1.68</td>
<td>2.02</td>
<td>2.42</td>
<td>2.70</td>
</tr>
<tr>
<td>42</td>
<td>1.30</td>
<td>1.68</td>
<td>2.02</td>
<td>2.42</td>
<td>2.70</td>
</tr>
<tr>
<td>43</td>
<td>1.30</td>
<td>1.68</td>
<td>2.02</td>
<td>2.42</td>
<td>2.70</td>
</tr>
<tr>
<td>44</td>
<td>1.30</td>
<td>1.68</td>
<td>2.02</td>
<td>2.41</td>
<td>2.69</td>
</tr>
<tr>
<td>45</td>
<td>1.30</td>
<td>1.68</td>
<td>2.01</td>
<td>2.41</td>
<td>2.69</td>
</tr>
<tr>
<td>46</td>
<td>1.30</td>
<td>1.68</td>
<td>2.01</td>
<td>2.41</td>
<td>2.69</td>
</tr>
<tr>
<td>47</td>
<td>1.30</td>
<td>1.68</td>
<td>2.01</td>
<td>2.41</td>
<td>2.68</td>
</tr>
<tr>
<td>48</td>
<td>1.30</td>
<td>1.68</td>
<td>2.01</td>
<td>2.41</td>
<td>2.68</td>
</tr>
<tr>
<td>49</td>
<td>1.30</td>
<td>1.68</td>
<td>2.01</td>
<td>2.40</td>
<td>2.68</td>
</tr>
<tr>
<td>50</td>
<td>1.30</td>
<td>1.68</td>
<td>2.01</td>
<td>2.40</td>
<td>2.68</td>
</tr>
</tbody>
</table>

We find $t_{49}^* = 1.68$. 

![T-distribution, df = 49](image)
Need to find the critical value $t^*_{df}$.

<table>
<thead>
<tr>
<th></th>
<th>one tail</th>
<th></th>
<th>two tails</th>
</tr>
</thead>
<tbody>
<tr>
<td>df</td>
<td>0.100</td>
<td>0.050</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>0.200</td>
<td>0.100</td>
<td>0.050</td>
</tr>
<tr>
<td>31</td>
<td>1.31</td>
<td>1.70</td>
<td>2.04</td>
</tr>
<tr>
<td>32</td>
<td>1.31</td>
<td>1.69</td>
<td>2.04</td>
</tr>
<tr>
<td>33</td>
<td>1.31</td>
<td>1.69</td>
<td>2.03</td>
</tr>
<tr>
<td>34</td>
<td>1.31</td>
<td>1.69</td>
<td>2.03</td>
</tr>
<tr>
<td>35</td>
<td>1.31</td>
<td>1.69</td>
<td>2.03</td>
</tr>
<tr>
<td>36</td>
<td>1.31</td>
<td>1.69</td>
<td>2.03</td>
</tr>
<tr>
<td>37</td>
<td>1.30</td>
<td>1.69</td>
<td>2.03</td>
</tr>
<tr>
<td>38</td>
<td>1.30</td>
<td>1.69</td>
<td>2.02</td>
</tr>
<tr>
<td>39</td>
<td>1.30</td>
<td>1.68</td>
<td>2.02</td>
</tr>
<tr>
<td>40</td>
<td>1.30</td>
<td>1.68</td>
<td>2.02</td>
</tr>
<tr>
<td>41</td>
<td>1.30</td>
<td>1.68</td>
<td>2.02</td>
</tr>
<tr>
<td>42</td>
<td>1.30</td>
<td>1.68</td>
<td>2.02</td>
</tr>
<tr>
<td>43</td>
<td>1.30</td>
<td>1.68</td>
<td>2.02</td>
</tr>
<tr>
<td>44</td>
<td>1.30</td>
<td>1.68</td>
<td>2.02</td>
</tr>
<tr>
<td>45</td>
<td>1.30</td>
<td>1.68</td>
<td>2.02</td>
</tr>
<tr>
<td>46</td>
<td>1.30</td>
<td>1.68</td>
<td>2.01</td>
</tr>
<tr>
<td>47</td>
<td>1.30</td>
<td>1.68</td>
<td>2.01</td>
</tr>
<tr>
<td>48</td>
<td>1.30</td>
<td>1.68</td>
<td>2.01</td>
</tr>
<tr>
<td>49</td>
<td>1.30</td>
<td>1.68</td>
<td>2.01</td>
</tr>
<tr>
<td>50</td>
<td>1.30</td>
<td>1.68</td>
<td>2.01</td>
</tr>
</tbody>
</table>

We find $t^*_{49} = 1.68$.

Since $\bar{x}_1 - \bar{x}_2 = 7.18 - 6.78 = 0.4$, we have

$$CI = 0.4 \pm 1.68 \times 0.258 = (-0.03, 0.83).$$
Need to find the critical value $t_{df}^*$.  

<table>
<thead>
<tr>
<th>df</th>
<th>one tail</th>
<th>0.100</th>
<th>0.050</th>
<th>0.025</th>
<th>0.010</th>
<th>0.005</th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>1.31</td>
<td>1.70</td>
<td>2.04</td>
<td>2.45</td>
<td>2.74</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>1.31</td>
<td>1.69</td>
<td>2.04</td>
<td>2.45</td>
<td>2.74</td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>1.31</td>
<td>1.69</td>
<td>2.03</td>
<td>2.44</td>
<td>2.73</td>
<td></td>
</tr>
<tr>
<td>34</td>
<td>1.31</td>
<td>1.69</td>
<td>2.03</td>
<td>2.44</td>
<td>2.73</td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>1.31</td>
<td>1.69</td>
<td>2.03</td>
<td>2.44</td>
<td>2.72</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>1.31</td>
<td>1.69</td>
<td>2.03</td>
<td>2.43</td>
<td>2.72</td>
<td></td>
</tr>
<tr>
<td>37</td>
<td>1.30</td>
<td>1.69</td>
<td>2.03</td>
<td>2.43</td>
<td>2.72</td>
<td></td>
</tr>
<tr>
<td>38</td>
<td>1.30</td>
<td>1.69</td>
<td>2.02</td>
<td>2.43</td>
<td>2.71</td>
<td></td>
</tr>
<tr>
<td>39</td>
<td>1.30</td>
<td>1.68</td>
<td>2.02</td>
<td>2.43</td>
<td>2.71</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>1.30</td>
<td>1.68</td>
<td>2.02</td>
<td>2.42</td>
<td>2.70</td>
<td></td>
</tr>
<tr>
<td>41</td>
<td>1.30</td>
<td>1.68</td>
<td>2.02</td>
<td>2.42</td>
<td>2.70</td>
<td></td>
</tr>
<tr>
<td>42</td>
<td>1.30</td>
<td>1.68</td>
<td>2.02</td>
<td>2.42</td>
<td>2.70</td>
<td></td>
</tr>
<tr>
<td>43</td>
<td>1.30</td>
<td>1.68</td>
<td>2.02</td>
<td>2.42</td>
<td>2.70</td>
<td></td>
</tr>
<tr>
<td>44</td>
<td>1.30</td>
<td>1.68</td>
<td>2.01</td>
<td>2.41</td>
<td>2.69</td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>1.30</td>
<td>1.68</td>
<td>2.01</td>
<td>2.41</td>
<td>2.69</td>
<td></td>
</tr>
<tr>
<td>46</td>
<td>1.30</td>
<td>1.68</td>
<td>2.01</td>
<td>2.41</td>
<td>2.69</td>
<td></td>
</tr>
<tr>
<td>47</td>
<td>1.30</td>
<td>1.68</td>
<td>2.01</td>
<td>2.41</td>
<td>2.69</td>
<td></td>
</tr>
<tr>
<td>48</td>
<td>1.30</td>
<td>1.68</td>
<td>2.01</td>
<td>2.41</td>
<td>2.68</td>
<td></td>
</tr>
<tr>
<td>49</td>
<td>1.30</td>
<td>1.68</td>
<td>2.01</td>
<td>2.40</td>
<td>2.68</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>1.30</td>
<td>1.68</td>
<td>2.01</td>
<td>2.40</td>
<td>2.68</td>
<td></td>
</tr>
</tbody>
</table>

We find $t_{49}^* = 1.68$.  

Since $\bar{x}_1 - \bar{x}_2 = 7.18 - 6.78 = 0.4$, we have  

$$CI = 0.4 \pm 1.68 \times 0.258 = (-0.03, 0.83).$$

We are 90% confident that babies born to non-smoking NC women are about $0.83$ to $-0.03$ lbs heavier than babies born to smoking NC women.
Your Turn!

Which of the following scenarios involve paired data?

1. Comparing students’ self-reports of “love for statistics” before and after E. Aamari’s class.
2. Assessing the gender-related salary gap by comparing salaries of men and women in the same randomly sampled positions at the same companies.
3. Comparing lung capacity changes in athletes before and after six weeks of training.
4. Assessing the claim that Uber is better than Lyft by dividing 70 random people into two groups of 35 and asking for their feedback on the one service they were assigned.
5. Exploring the average attractiveness of husbands and wives in couples who own a yacht.

Answer:

1. Paired. The linkage is the student.
2. Paired. The linkage is the common job.
3. Paired. The linkage is the athlete.
4. No paired. Paired data would be people trying both.
5. Paired. The linkage is marriage.
Your Turn!

Which of the following scenarios involve paired data?
1. Comparing students’ self-reports of “love for statistics” before and after E. Aamari’s class.
2. Assessing the gender-related salary gap by comparing salaries of men and women in the same randomly sampled positions at the same companies.
3. Comparing lung capacity changes in athletes before and after six weeks of training.
4. Assessing the claim that Uber is better than Lyft by dividing 70 random people into two groups of 35 and asking for their feedback on the one service they were assigned.
5. Exploring the average attractiveness of husbands and wives in couples who own a yacht.

Answer:
1. Paired. The linkage in the student.
2. Paired. The linkage is the common job.
3. Paired. The linkage is the athlete.
4. No paired. Paired data would be people trying both.
5. Paired. The linkage is marriage.
Let’s Get Huge!

Holding other variables constant, chickens were fed 6 different types of feeds to make them huge for American consumers. Do these data suggest the average weights of chickens on meatmeal and casein are different?

<table>
<thead>
<tr>
<th>Feed</th>
<th>Mean</th>
<th>SD</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>casein</td>
<td>323.58</td>
<td>64.43</td>
<td>12</td>
</tr>
<tr>
<td>horsebean</td>
<td>160.20</td>
<td>38.63</td>
<td>10</td>
</tr>
<tr>
<td>linseed</td>
<td>218.75</td>
<td>52.24</td>
<td>12</td>
</tr>
<tr>
<td>meatmeal</td>
<td>276.91</td>
<td>64.90</td>
<td>11</td>
</tr>
<tr>
<td>soybean</td>
<td>246.43</td>
<td>54.13</td>
<td>14</td>
</tr>
<tr>
<td>sunflower</td>
<td>328.92</td>
<td>48.84</td>
<td>12</td>
</tr>
</tbody>
</table>

Remark: Given the small sample sizes and skew seen in the boxplot of meatmeal and casein, we should not proceed with inference. We probably don’t meet the Nearly Normal condition needed for each sample.
Let’s Get Huge!

Holding other variables constant, chickens were fed 6 different types of feeds to make them huge for American consumers. Do these data suggest the average weights of chickens on meatmeal and casein are different?

**Remark:** Given the small sample sizes and skew seen in the boxplot of meatmeal and casein, we should not proceed with inference. We probably don’t meet the Nearly Normal condition needed for each sample.
Do inference on the difference of mean weights of chickens on horsebean and linseed. Create a 95% CI and run a Hypothesis Test with $\alpha = 0.05$. 

Our point estimate for the (unknown) parameters $\mu_L - \mu_H$ is $\bar{x}_L - \bar{x}_H = 218.75 - 160.20 = 58.55$ grams. 

Our sampling distribution is $t_9$ with $SE = \sqrt{52.24^2/12 + 38.63^2/10} \approx 19.41$. 

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>SD</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>casein</td>
<td>323.58</td>
<td>64.43</td>
<td>12</td>
</tr>
<tr>
<td>horsebean</td>
<td>160.20</td>
<td>38.63</td>
<td>10</td>
</tr>
<tr>
<td>linseed</td>
<td>218.75</td>
<td>52.24</td>
<td>12</td>
</tr>
<tr>
<td>meatmeal</td>
<td>276.91</td>
<td>64.90</td>
<td>11</td>
</tr>
<tr>
<td>soybean</td>
<td>246.43</td>
<td>54.13</td>
<td>14</td>
</tr>
<tr>
<td>sunflower</td>
<td>328.92</td>
<td>48.84</td>
<td>12</td>
</tr>
</tbody>
</table>
Do inference on the difference of mean weights of chickens on horsebean and linseed. Create a 95% CI and run a Hypothesis Test with $\alpha = 0.05$.

Our point estimate for the (unknown) parameters $\mu_L - \mu_H$ is

$$\bar{x}_L - \bar{x}_H = 218.75 - 160.20 = 58.55 \text{ grams}.$$
Do inference on the difference of mean weights of chickens on horsebean and linseed. Create a 95% CI and run a Hypothesis Test with $\alpha = 0.05$.

Our point estimate for the (unknown) parameters $\mu_L - \mu_H$ is

$$\bar{x}_L - \bar{x}_H = 218.75 - 160.20 = 58.55 \text{ grams}.$$ 

Our sampling distribution is $t_9$ with

$$SE = \sqrt{\frac{52.24^2}{12} + \frac{38.63^2}{10}} \approx 19.41.$$
From the table, $t^*_9 = 2.262$, so

$$CI = 58.55 \pm 2.262 \times 19.41$$

$$= (14.64, 102.46)$$

Notice that 0 isn’t in this interval. So the difference in parameter values is unlikely to be 0.
From the table, \( t^*_9 = 2.262 \), so

\[
CI = 58.55 \pm 2.262 \times 19.41 = (14.64, 102.46)
\]

Notice that 0 isn’t in this interval. So the difference in parameter values is unlikely to be 0. Under \( H_0 : \mu_L - \mu_H = 0 \), we get:

\[
T = \frac{58.55 - 0}{19.4} \approx 3.018.
\]

With a two-sided alternative, the \( p \)-value is \( p = P(|T_9| > 3.018) \).

From the table, \( p \) satisfies

\[
0.01 \leq p \leq 0.02
\]
From the table, $t_9^* = 2.262$, so

$$CI = 58.55 \pm 2.262 \times 19.41$$

$$= (14.64, 102.46)$$

Notice that 0 isn’t in this interval. So the difference in parameter values is unlikely to be 0. Under $H_0: \mu_L - \mu_H = 0$, we get:

$$T = \frac{58.55 - 0}{19.4} \approx 3.018.$$  

With a two-sided alternative, the $p$-value is $p = P(|T_9| > 3.018)$. From the table, $p$ satisfies $0.01 \leq p \leq 0.02$.

The $p$-value $p \leq 0.02 < 0.05$ leads us to reject the null $H_0: \mu_L - \mu_H = 0$ in favor of $H_A: \mu_L - \mu_H \neq 0$. (As already guessed with the CI)