Math 11
Calculus-Based Introductory Probability and Statistics

Eddie Aamari
S.E.W. Assistant Professor

 eaamari@ucsd.edu
 math.ucsd.edu/~eaamari/
 AP&M 5880A

Today:
• The Normal distribution
• z-Score, z-Tables
The Normal Distribution

The density function of the normal distribution is:

\[ f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x - \mu)^2}{2\sigma^2}}, \]

where \( \mu \) is the mean, and \( \sigma \) is the standard deviation.

Notation: \( X = N(\mu, \sigma) \).

Other name: Gaussian distribution
Importance of the Normal Distribution

The Normal model is the most important continuous random variable in all of modern statistics.

Roughly speaking, this comes from the fact that any time some quantity is the combination of many independent factors, then this quantity will follow a normal model.
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Examples:

- Human heights in the US (Female: $\mu = 65$ in, $\sigma = 3.5$ in)
- Diastolic blood pressure ($\mu = 77$ mm Hg, $\sigma = 5.5$ mm Hg)
- IQ scores ($\mu = 100$, $\sigma = 15$)
Normal Distribution: Example

Suppose the height of US women is normally distributed with a mean $\mu = 65$ inches and standard deviation $\sigma = 3.5$ inches. What is the probability the next woman you see has a height over 72 inches?
Normal Distribution: Example

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What is the probability the next woman you see has a height over 72 inches?
Let $X = N(65, 3.5)$. We want

$$P(X \geq 72) = \int_{72}^{\infty} \frac{1}{3.5\sqrt{2\pi}} e^{-\frac{(x-65)^2}{2(3.5)^2}} dx$$
Normal Distribution: Example

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\[
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\]

\[= ...\]

That’s a hard integral! This is the first example of a distribution where probabilities cannot be found by hand.
Technology and the $z$-table

**Technology:**

Graph $\gg$ Probability Distribution Plot $\gg$ View Probability

There is a 2.275% chance a random woman is over 6 feet tall.
Technology and the $z$-table

**Technology:**

![Graph interface](image1)

**The $z$-Table:**

This is a way to look up the areas under part of a normal curve and get the answer 0.02275.

![Distribution plot](image2)

There is a 2.275% chance a random woman is over 6 feet tall.
Technology and the $z$-table

**Technology:**

This image shows a probability distribution plot with a shaded area indicating the probability of a random woman being over 6 feet tall. The area under the curve between 65 and 72 is 0.02275.

**The $z$-Table:**

This is a way to look up the areas under part of a normal curve and get the answer 0.02275.

**Trouble:**

We don’t want a different table for every possible normal curve (recall that the mean can be any number, and the standard deviation can be any positive number!) We need a way to convert any situation modelled by a normal curve into some standard setup.
You are in charge of admissions at UCSD. One applicant took the SAT and got a 1775. Another took the ACT and got 27. Which would you admit?

$z$-score
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z-score

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The z-score of a data point $y$ from a dataset is $\frac{y - \bar{y}}{s_y}$. 
z-score

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The z-score of a data point \( y \) from a dataset is \( \frac{y - \bar{y}}{s_y} \).

The z-score:
- Is a unitless idea (units in numerator and denominator cancel)
- Tells you how many standard deviations above the mean some piece of data is.
You see on Google that the SAT has mean 1500 with SD 250, and the ACT has mean 20.8 with SD 4.8. Which do you admit, the 1775 SAT or 27 ACT?
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\[ z_{SAT} = \frac{1775 - 1500}{250} = 1.1 \quad \quad \quad z_{ACT} = \frac{27 - 20.8}{4.8} \approx 1.29. \]
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Assuming the two tests are equally difficult, you’d rather admit the ACT person.
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\textit{z}-scores provide a single “ruler idea” to measure all phenomena, erasing the effect of units.

The \textit{z}-score says how extreme a data point is relative to its own data set.
**$z$-score**

Let’s take a data set and find the $z$-score for every data point.

$$\mu_{SAT} = 1500, \quad \sigma_{SAT} = 250$$

\[ z_x = \frac{x - \mu_{SAT}}{\sigma_{SAT}} \]
Let’s take a data set and find the $z$-score for every data point.

$$\mu_{SAT} = 1500, \quad \sigma_{SAT} = 250$$

It appears:

- The new histogram is similar
- The new mean is 0.
- The new standard deviation is 1.
z-Score Summary

- $z$-scores allow us to compare two data points from different data sets (with different centers and spreads) and get a sense for which datum is more extreme relative to its own data set.
**z-Score Summary**

- *z*-scores allow us to compare two data points from different data sets (with different centers and spreads) and get a sense for which datum is more extreme relative to its own data set.

- *z*-scores allow us to rescale a given data set so it has mean 0 and standard deviation 1. In the case of probability models, this allows us to think about whole families of curves using a single “standardized” model.
By rescaling data with the $z$-score, we turn all Normal models $N(\mu, \sigma)$ into a single one: the **Standard Normal Model** $N(0, 1)$.
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Any question in the original setting can be reframed as a question on the standard Normal model $N(0, 1)$. 

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**Standard Normal Model $N(0,1)$**

![Standard Normal Model Graph](image)
All Roads Lead To ... The Standard Normal Model

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Any question in the original setting can be reframed as a question on the standard Normal model $N(0, 1)$.

If we understand $N(0, 1)$, we understand all the Normal models.
Find the percentage of students that has an SAT score below 1800.

A priori, we could find this probability by finding the area under the density curve using an integral. For normal distributions, this is too difficult to do by hand.
One Question, Many Ways to Solve

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Option 1: Original setting + technology. $X = N(1500, 250)$, and we want $P(X \leq 1800)$. 
Doing This in Minitab

Go to Graph » Probability Distribution Plot » View Probability

[Diagram of Probability Distribution Plots dialog box]
Doing This in Minitab

Go to Graph » Probability Distribution Plot » View Probability

Set up the distribution you want (here: Normal with mean 1500 and SD 250), then click on “Shaded Area” tab. Specify the area you are trying to find.
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Go to Graph » Probability Distribution Plot » View Probability

Set up the distribution you want (here: Normal with mean 1500 and SD 250), then click on “Shaded Area” tab. Specify the area you are trying to find.

You get a nice plot with the answer displayed. Note that this does NOT require converting 1800 to a z-score.
Option 2: Standardized setting + tables.

We notice that

\[ P(X \leq 1800) = P \left( \frac{X - 1500}{250} \leq \frac{1800 - 1500}{250} \right) = P(Z \leq 1.2). \]
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We want the area less than \( z = 1.20 \) under the Standard Normal.
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We want the area less than $z = 1.20$ under the Standard Normal.

Any question about Normal curves can be converted to an equivalent question on the standard normal curve. We just need a lookup table of “areas under the curve” for the standard normal!

On tests, you won’t have access to Minitab, so you will have to use this approach.
The values in the table are all areas (probabilities) The number along the top and left side are the $z$ – value broken into its two parts.
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The values in the table are all areas (probabilities) The number along the top and left side are the $z - value$ broken into its two parts.

Here, $z = 1.20 = 1.2 + 0.00$, so we find $P(Z \leq 1.20) \approx 0.8849$. 

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Disneyland

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Let $X = N(55, 6)$. We want $P(X \geq 44)$. 

Minitab gives use $p = 0.9666$.

So, 96.7% of 10-year-olds can ride the Space Mountain ride at Disneyland.
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![Distribution Plot](image-url)
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Writing \( Z = \frac{X - 55}{6} \), we see that we want
\[ P(X \geq 44) = P \left( Z \geq \frac{44 - 55}{6} \right) = P \left( Z \geq -1.833 \right) = 1 - P(\text{Normal}) \]
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$$P(X \geq 44) = P \left( Z \geq \frac{44 - 55}{6} \right)$$

$$= P(Z \geq -1.83)$$

$$= 1 - P(Z < -1.83)$$
Practice With Tables

We read $P(Z < -1.83) \approx 0.0336$, so

$$P(X \geq 44) \approx 1 - P(Z < -1.83) \approx 1 - 0.0336 = 0.9664.$$
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**Remark:** On these types of problems, never worry about rounding issues or slight difference in answers, we just want approximations.
Sleep Time

The sleep times (in hours) of American men on a weekday are well-modeled by $N(6.9, 1.5)$. What percentage of American men are within 1 standard deviation of the mean sleep time?
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Let $X = N(6.9, 1.5)$. We want $P(5.4 \leq X \leq 8.4)$. 

About 68% of American men are within one $\sigma$ of the mean.
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$$P(5.4 \leq X \leq 8.4) = P(X \leq 8.4) - P(X < 5.4).$$
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About 68% American men are within one $\sigma$ of the mean.
The $68 - 95 - 99.7\%$ (Approximate) Rule

- About 68% of the data values are within 1 SD of the mean.
- About 95% are within 2 SDs.
- About 99.7% are within 3 SDs.
The 68 – 95 – 99.7% (Approximate) Rule

This holds for any data set that is normally distributed.

- About 68% of the data values are within 1 SD of the mean (blue).
- About 95% are within 2 SDs (blue and green).
- About 99.7% are within 3 SDs (blue, green, and yellow).
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A Question using the 68 − 95 − 99.7% Rule

What percentage of students score above a 1250 on the SAT? \((\mu = 1500, \sigma = 250)\)
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So, the area to the left of -1 SD is \(32/2 = 16\%\) (by symmetry)
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The desired percentage is $100 − 16 = 84\%$. 
Percentiles and “Going Backwards”

For any $x$ value (or $z$-score, if you convert to a standard normal), the **percentile** is simply the area to the left of this value.
Percentiles and “Going Backwards”

For any \( x \) value (or \( z \)-score, if you convert to a standard normal), the percentile is simply the area to the left of this value.

Example: the value \( x = 1800 \) on the SAT is about the 88th percentile.

This means you scored higher than 88% of people who took the SAT.
Percentiles and “Going Backwards”

Suppose a college only takes students who reach the 99th percentile (or better) on the SAT. What cutoff must you attain?
Percentiles and “Going Backwards”

Suppose a college only takes students who reach the 99th percentile (or better) on the SAT. What cutoff must you attain?

We need this area to be 0.99.

What must this be?

Probability: 0.99

You must score at or above a 2082 to get into this college!
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As a New Year’s resolution, you decide to get more sleep than 93% of American men. What is the least amount you can sleep per night? We remember that sleep is modelled by $N(6.9, 1.5)$.

We find the area in the table that gives 93%:
“Going Backwards” with Tables

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We find the area in the table that gives 93%:

<table>
<thead>
<tr>
<th>$Z$</th>
<th>0.00</th>
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We find the area in the table that gives 93%: $z_{93\%} = 1.48$

Since the initial model is $X = N(6.9, 1.5)$, we get

$$z_{93\%} = \frac{x_{93\%} - 6.9}{1.5}$$

which we solve to get

$$x_{93\%} = 1.5 \times z_{93\%} + 6.9 = 9.12$$ hours/night.
But Wait! Are My Data Really Normal?

1) The human eye: Does the histogram look unimodal and symmetric?
But Wait! Are My Data Really Normal?

Two Tests for Normality:

1) The human eye:
   Does the histogram look unimodal and symmetric?
But Wait! Are My Data Really Normal?

Two Tests for Normality:

2) Use a “probability plot”. Minitab: Graph » Probability Plot
If the data fall on a straight line, that implies normality. If not, your data are non-normal.
Joe Bob is totally average in every way. What will his $z$-score be when he takes the SAT?

1. 1500
2. 0
3. 1
4. -1
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Answer: 2., by definition of the \( z \)-score!
A recent stat test had a mean of 80% with a SD of 3%. What test score has a \( z \)-score -3?

1. 89%
2. 89
3. 71%
4. 71
A recent stat test had a mean of 80% with a SD of 3%. What test score has a $z$-score -3?

1. 89%
2. 89
3. 71%
4. 71

Answer: 3. Notice that \((71\% - 80\%)/3\% = -3\) (no unit)
The 89% has a $z$-score of +3.
Who was better in his sport?

- Michael Phelps, swimming, 27 olympic medals
  (mean olympic medals won by swimmers: 0.3, SD: 0.1)
- Barry Bonds, baseball, 762 home runs
  (mean home runs by baseball player: 4, SD: 3)

1. Phelps
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1. Phelps
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Answer: 1., since Phelps $z$-score is $(27-0.3)/0.1 = 267$, and Bonds’ is $(762-4)/3 = 252.7$, and $267 > 252.7$. 
Practice

The median on a test is 60. If the teacher halves everyone’s score and adds 50, what is the new median?

1. 55
2. 110
3. 30
4. 80

Answer: 4., since the median undergoes all the transformation the data does. So $60/2 + 50 = 80$ is the new median. (This reasoning would also apply to the mean)
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(This reasoning would also apply to the mean)
The SD of temperature in a city is 10 degrees Celsius (C). What is the SD if the data are measured in Fahrenheit (F)?

Recall that $F = \left(\frac{9}{5}\right)C + 32$

1. 50° F
2. 18° F
3. 10° F
4. −3° F
The SD of temperature in a city is 10 degrees Celsius (C). What is the SD if the data are measured in Fahrenheit (F)? Recall that $F = \left(\frac{9}{5}\right)C + 32$

1. $50^\circ$ F
2. $18^\circ$ F
3. $10^\circ$ F
4. $-3^\circ$ F

Answer: 2., since $\left(\frac{9}{5}\right) \times 10 = 18$.
(SD’s are only affected by scaling, not shifting)
Practice

Billy takes the weight of everyone in his class and converts them to $z$-scores. What is the mean of the data when written as $z$-scores?

1. Cannot determine with given info
2. 0
3. 1
4. -1

Answer: 2., by definition of $z$-scores!!
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We have \[ \text{New Data} = \text{Old data} - \text{mean} \cdot \text{SD}. \] The SD ignores shifts like subtracting the mean. To find the new SD, we just apply the scaling in the formula (division by the old SD). So the new SD is \[ \text{SD/SD} = 1. \]
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Suppose you data set collects random variables $X$ with mean $\mu$ and standard deviation $\sigma$. 
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Also,

$$SD(Z) = SD\left(\frac{X - \mu}{\sigma}\right) = \frac{1}{\sigma}SD(X - \mu) = \frac{1}{\sigma}SD(X)$$

$$= \frac{1}{\sigma} = 1.$$