The remarks are intended to enhance your understanding.
You do not have to hand in your answers to them.

11–16 Use the given transformation to evaluate the integral.

12. $\iint_R (4x+8y)\,dA$, where R is the parallelogram with vertices $(-1,3)$, $(1,-3)$, $(3,-1,)$ and $(1,5)$;
$x = \frac{1}{4}(u+v)$, $y = \frac{1}{4}(v-3u)$.
Remark: How does this improve the boundary of the region?

14. $\iint_R (x^2 - xy + y^2)\,dA$, where R is the region bounded by the ellipse $x^2 - xy + y^2 = 2$;
$x = \sqrt{2}\,u - \sqrt{2/3}\,v$, $y = \sqrt{2}\,u + \sqrt{2/3}\,v$
Remark: What does this do to the boundary and to the integrand? Do you think this would still be a good idea if the integrand were $x^2 + y^2$?

16. $\iint_R y^2\,dA$, where R is the region bounded by the curves $xy = 1$, $xy = 2$, $xy^2 = 1$, $xy^2 = 2$;
$u = xy$, $v = xy^2$. Illustrate the region R. (You may use a calculator.)

S1. Evaluate the improper integral $\iint x^2 e^{-(x^2+y^2)} dA$ over the entire xy-plane.

S2. A sphere of radius R is bounded by the surface $x^2 + y^2 + z^2 = R^2$. By transforming $\iiint_R dV$ to spherical coordinates, evaluate the volume of the sphere.

17. (a) Evaluate $\iiint_E dV$, where E is the solid enclosed by the ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$.
Use the transformation $x = au$, $y = bu$, $z = cu$.

(b) The Earth is not a perfect sphere; rotation has resulted in a flattening at the poles. So the shape can be approximated by an ellipsoid with $a = b = 6378$ km and $c = 6356$ km. Use part (a) to estimate the volume of the Earth.

18. Evaluate $\iiint_E x^2 y\;dV$, where E is the solid of Exercise 17(a).