

Name \_\_\_\_\_ ID No. \_\_\_\_\_

There are 125 points total. (So first exam is 20% and this is 25%.)

1. (45 pts.) Indicate whether true or false. Beware of guessing:

correct answer +5pts.      incorrect answer -3pts.      no answer 0pts

- (a) \_\_\_ Every finite set of strings is a CFL.
  - (b) \_\_\_ The language  $\{a^n b^n c^n | n > 0\}$  can be recognized by a (1-stack) PDA.
  - (c) \_\_\_ A PDA with two stacks can recognize more languages than a standard 1-stack PDA.
  - (d) \_\_\_ If  $L$  is Turing-decidable, then  $\bar{L}$  is also Turing-decidable.
  - (e) \_\_\_ A Turing machine with two tapes can recognize more languages than a standard 1-tape Turing machine.
  - (f) \_\_\_ The language  $\{a^n b^n c^n d^n | n > 0\}$  is Turing-recognizable.
  - (g) \_\_\_  $L$  is Turing-decidable when  $L$  is the set of strings of digits that represent primes; that is,  $L = \{2, 3, 5, 7, 11, 13, \dots\}$ . ( $n$  is a prime if its only positive integer divisors are itself and 1.)
  - (h) \_\_\_ There exists a Turing machine which can decide if two DFAs are equivalent; that is, whether or not they recognize the same language.
  - (i) \_\_\_ There exists a Turing machine  $M$  which can decide if a Turing machine will loop on a given input; that is,  $M$ 's input is a description of a machine, say  $T$ , and a string, say  $w$ , and  $M$  accepts the input if  $T$  does loop on  $w$  and  $M$  rejects the input if  $T$  does not loop on  $w$ .
2. (25 pts.) Prove that, if  $L$  and  $M$  are CFLs, then so is  $L \cup M$ .

**MORE**

3. (30 pts.) Let  $L = \{a^nbc^n \mid n \geq 0\}$ .

(a) Construct a context free grammar to generate the language.

(b) Construct a PDA to recognize the language.

4. (25 pts.) Suppose that both  $L$  and  $\bar{L}$  are Turing-recognizable. Either (a) prove that  $L$  must be Turing-decidable, or (b) give an example of such an  $L$  which is not Turing-decidable.

**END**