• DO NOT begin working, or even open this packet, until instructed to do so.

• You should be in your assigned seat, unless instructed otherwise by Ed or one of the TAs.

• Enter all requested information on the top of this page, and put your name on the top of every page, in case the pages become separated.

• You may use a two-sided page of notes on this exam.

• You may not use your books, additional notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

• Organize your work, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.

• Mysterious or unsupported answers will not receive full credit. Unless otherwise directed in the statement of the problem, a correct answer, unsupported by calculations, explanation, or algebraic work will receive little or no credit. An incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

• Unless the problem says otherwise, a complete answer should have no integrals in it, definite or indefinite. On the other hand, implicit solutions are ok.

• Some initial value problems might not have global solutions. Local solutions (that is, solutions valid near the initial condition, but possibly not elsewhere) are acceptable for all initial value problems.

DO NOT turn this page until instructed to do so
1. (10 points) Solve the initial value problem.

\[ y' - 3y + e^{2x} = 0, \quad y(0) = 1 \]
2. (10 points) Solve the initial value problem.

\[ x \cos(y)y' + (\sin(y) + 1) = 0, \quad y(0) = 1 \]
3. (10 points) Find the general solution.

\[ y'' + y = \frac{1}{2 + \sin(x)}, \quad x > 1 \]

(Thanks to A. K. Lal for this example)
4. (5 points) Find a constant $2 \times 2$ matrix $P$ such that any solution $\vec{x}(t)$ to the equation $\vec{x}' = P\vec{x}$ goes in perfect circles around $\vec{0}$ as $t$ varies.

5. (5 points) Find a function $f(u)$ such that

- $y = 3$ and $y = 5$ are both solutions to the equation $y' = f(y)$.

- If $y(x)$ is a solution to $y' = f(y)$ and $3 < y(0) < 5$, then $\lim_{x \to \infty} y(x) = 4$. 
6. (10 points) Find the general solution to this system of differential equations:

\[
x_1' = 3x_1 + 4x_2 \\
x_2' = -x_1 + 7x_1
\]
7. Suppose that \( y = \sum_{k=0}^{\infty} a_k x^k \) is a solution to the initial value problem

\[ y'' + x^2 y = e^x \quad y(0) = 1, \ y'(0) = 2 \]

(a) (7 points) Express \( a_k \) in terms of \( a_{k-1}, a_{k-2}, \ldots \). Your formula need only work for large values of \( k \).

(b) (3 points) Find the degree-4 Taylor polynomial of \( y \) at \( x = 0 \).
8. Let \( h(t) = \begin{cases} 
0 & t < 1 \\
1 & 1 \leq t < 4 \\
0 & 4 \leq t 
\end{cases} \)

Compute

\[ \mathcal{L}\{ \sin(t) \ast (e^{-t}t + \delta(t)) \ast h(t) \} \]

There should be no integrals in your answer.
9. (10 points) Consider the initial value problem

\[ y'' + 4y' + 3y = f(t), \quad y(0) = y'(0) = 0 \]

Find a function \( g(t) \) such that the solution to this initial value problem is given by \( y(t) = g(t) \ast f(t) \).
10. (10 points) Find the general solution to

\[ y'' - y = \delta(t - 1) \]
Scratchwork goes here
### Table of Laplace Transforms

\[
\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t) \, dt
\]

\[
\mathcal{L}\{1\} = \frac{1}{s}
\]

\[
\mathcal{L}\{t\} = \frac{1}{s^2}
\]

\[
\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}
\]

\[
\mathcal{L}\{\sin(\omega t)\} = \frac{\omega}{s^2 + \omega^2}
\]

\[
\mathcal{L}\{\cos(\omega t)\} = \frac{s}{s^2 + \omega^2}
\]

\[
\mathcal{L}\{\sinh(\omega t)\} = \frac{\omega}{s^2 - \omega^2}
\]

\[
\mathcal{L}\{\cosh(\omega t)\} = \frac{s}{s^2 - \omega^2}
\]

\[
\mathcal{L}\{\delta(t)\} = 1
\]

\[
\mathcal{L}\{e^{-at}f(t)\} = \mathcal{L}\{f(t)\}(s + a)
\]

\[
\mathcal{L}\{u(t-a)f(t-a)\} = e^{-at}\mathcal{L}\{f(t)\}
\]

\[
\mathcal{L}\{f'(t)\} = s\mathcal{L}\{f(t)\} - f(0)
\]

\[
\mathcal{L}\{f''(t)\} = s^2\mathcal{L}\{f(t)\} - sf(0) - f'(0)
\]

\[
\mathcal{L}\left\{ \int_0^t f(p) \, dp \right\} = \frac{1}{s}\mathcal{L}\{f(t)\}
\]

\[
\mathcal{L}\{f(t) * g(t)\} = \mathcal{L}\{f\}\mathcal{L}\{g\}
\]