1. Let $V$ be the vector space of $1 \times 2$ vectors (also known as “row vectors”), with addition and scalar multiplication given by the usual addition and scalar multiplication operations on matrices. Note that if $B \in V$ and $A$ is a $2 \times 2$ matrix, then $BA$ is a $1 \times 2$ matrix, so it is another element of $V$. $V$ has a basis $\mathcal{B} = \{[1, 0], [0, 1]\}$.

Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Find a matrix $A'$ such that, for any $B \in V$,

$$A'[B]_\mathcal{B} = [BA]_\mathcal{B}$$

2. Let $V$ be the vector space of functions $\mathbb{R} \to \mathbb{R}$ and let $H = \text{span}\{\sin(x), \cos(x)\}$.

Let $\mathcal{B} = \{\sin(x), \cos(x)\}$.

(a) Explain why $\mathcal{B}$ is a basis for $H$.

(b) For which values of $a, b, c$ is $\{a \sin(x) + b \cos(x), c \sin(x) + 7 \cos(x)\}$ a basis of $H$? Hint: Figure out when

$$\{[a \sin(x) + b \cos(x)]_\mathcal{B}, [c \sin(x) + 7 \cos(x)]_\mathcal{B}\}$$

is a basis of $\mathbb{R}^2$. 

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