(1) Define the following objects
(a) a commutative ring
(b) a zero divisor
(c) an integral domain
(d) the direct sum of $n$ commutative rings
(e) the characteristic of a commutative ring
(f) an ideal
(g) a PID
(h) the quotient of a commutative ring by an ideal
(i) a prime ideal
(j) a maximal ideal
(k) the field of fractions of an integral domain
(l) a field extension
(m) an algebraic element of a field extension
(n) a transcendental element of a field extension
(o) the minimal polynomial of an algebraic element
(p) a finite field extension
(q) the degree of a finite field extension
(r) an algebraic field extension
(s) a transcendental field extension
(t) a simple field extension
(u) an element of $\mathbb{R}$ constructible over a subfield $F$ of $\mathbb{R}$
(v) the splitting field of a polynomial over a given field

(2) Find all the idempotents and all the units of the rings $\mathbb{Z} \oplus \mathbb{Z}$ and $\mathbb{Z}_{10} \oplus \mathbb{Z}_{12}$.

(3) Let $E$, $F$ be two fields and $\phi : E \to F$ a ring homomorphism. Prove that $\phi$ is an isomorphism if and only if $\phi$ is surjective.

(4) Prove that the direct sum of two nonzero rings is never an integral domain.

(5) Given two positive integers $m$ and $n$, find all the ring homomorphisms from $\mathbb{Z}_m$ to $\mathbb{Z}_n$.

(6) What is the characteristic of the ring $\mathbb{Z}_m \oplus \mathbb{Z}_n$?

(7) Suppose $R$ is an integral domain of characteristic $p > 0$. Prove by induction that, for all $a, b \in R$ and all $n \in \mathbb{N}$, $(a + b)^p = a^p + b^p$.

(8) Assume $R$ is a commutative ring and that $R$ is either finite or a finite dimensional vector space over a field. Prove that every prime ideal in $R$ is maximal.

(9) Let $K \subset L \subset M$ be extensions of fields (we call this a tower). Prove that $M$ is algebraic over $K$ if and only if $M$ is algebraic over $L$ and $L$ is algebraic over $K$.

(10) For any positive integers $a, b$, prove that $\mathbb{Q}(\sqrt{a}, \sqrt{b}) = \mathbb{Q}(\sqrt{a} + \sqrt{b})$.

(11) For which integers $n$ between 3 and 10 is a regular $n$-gon constructible?

(12) Use the problems from your homework 7 to prove that there are exactly 8 distinct automorphisms of the splitting field $\mathbb{Q}(\sqrt{2}, i)$ of $x^4 - 2$ over $\mathbb{Q}$. 