We can keep doing this until \( n = 0 \).

Now we have \( P(x, y, z) = x y^m + y R(x, y, z) \)

since \( P(0, 0, 1) = 0 \), we have \( n = 0 \)

so \( P(x, y, z) = y R(x, y, z) \)

\( Q(x, y, z) = Q(x, 0, z) + y S(x, y, z) \)

Now \( I_y(P, Q) = I_y(y, Q) + I_y(R, Q) \) by (iv)

\( \) by (vi)

\( I_y(y, Q(x, 0, z)) \)

\( Q(x, 0, z) \) is homogeneous in 2 variables, so it is a product of linear factors in \( x \) and \( z \) (they are independent of \( y \))

so \( I_y(y, Q(x, 0, z)) = \deg Q \) by (v) and (iv)

so \( k = I_y(P, Q) = \deg Q + I_y(R, Q) \)

\( \deg Q > 0 \) so \( I_y(R, Q) < k \)
By the induction hypothesis, \( I_P(R, Q) \) can be computed from (i), \( \ldots \), (vi). So \( I_P(P, Q) \) can also be computed.

**Proof of Bezout's theorem:**

Choose coordinates so that A, B, C hold.

Then by Lemmas 3.4, 3.7, \( R_{P, Q} = \prod_{i=1}^{e_i} (c_i y - b_i z) \),

where \((b_i, c_i)\) and \((b_j, c_j)\) are not proportional for \(i \neq j\).

Recall: \( \forall i \in R_{P(x_i, b_i, c_i), Q(x_i, b_i, c_i)} = 0 = R_{P, Q}(b_i, c_i) \)

\( \implies \exists! a_i : \forall i \in R_{P(a_i, b_i, c_i), Q(a_i, b_i, c_i)} = 0 \)

We have uniqueness of \( a_i \) because otherwise, we would have \((a_i, b_i, c_i) \& (a'_i, b_i, c_i) \in C \cap D\) are collinear with \((1, 0, 0) = (a_i - a'_i, 0, 0) \)

if \( a_i \neq a'_i \).
We saw in the proof of Theorem 3.18 that
\[ I_{(c_{i}, e_{i}, c_{i})} (C, D) = e_{i} \]

By Lemma 3.7, \[ \deg R_{p, q} = m_{u} \sum_{i=1}^{k} e_{i} \]
Also, \[ \deg R_{p, q} = e_{1} + \cdots + e_{k} = \sum_{i=1}^{k} e_{i} \]
So, \[ \sum_{P \in C \cap D} I_{P} (C, D) = \sum_{i=1}^{k} e_{i} = \deg R_{p, q} = m_{u} \]

Lemma 3.24: If \( P \in C \cap D \) is a singular point of \( C \), then \( I_{P} (C, D) > 1 \).