MATH 106

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Please explain or prove all your assertions and show your work. Please state all the definitions, propositions, theorems, lemmas that you use precisely.

Please make sure to review all definitions, statements of theorems, proofs done in class, practice problems and homework problems. The exercises below are complementary to the rest of the material and do not replace the book, lectures, homework problems, etc.

Although homework 7 is due after the test, its material is covered by the test: please make sure to do homework 7 before the midterm.

Good luck!

(1) Let C be an irreducible conic in $\mathbb{P}^2$. Prove that there exists a coordinate system in which the equation of C is $z^2 - xy = 0$.

(2) Let C be a non-singular cubic in $\mathbb{P}^2$. Prove that there exists $\lambda \in \mathbb{C} \setminus \{0, 1\}$ and a coordinate system in which the equation of C is $y^2z - x(x - z)(x - \lambda z) = 0$.

(3) Let C be an irreducible singular cubic in $\mathbb{P}^2$. Prove that there exists a coordinate system in which the equation of C is either $y^2z - x^3 = 0$ or $y^2z - x^2(x - z) = 0$. What are the singularities of C and what are their multiplicities? Are they ordinary? What are the points of inflection of C?

(4) Define the intersection multiplicity of two curves at a point p of $\mathbb{P}^2$.

(5) Prove that the intersection multiplicity exists and is unique.

(6) Prove Bézout's theorem.

(7) Prove that $I_p(C, D) = 1$ if and only if C and D are both smooth at p and their tangent lines at p are distinct.

(8) State and prove Pascal's theorem.

(9) Let C be a projective curve with equation $P = 0$ and p a point of C. Prove that there exists a line meeting C at p with multiplicity at least 3 if and only if $\mathcal{H}_P(p) = 0$.