

MATH 120A

Elham Izadi

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Please justify all your assertions and state all the definitions, propositions, theorems, lemmas that you use precisely. Also make sure to review other definitions, theorems and proofs that we did, homework problems and the relevant sections of the book. Good luck!

- (1)
 - (a) Give the definition of a piecewise smooth arc or contour.
 - (b) Give the definition of a simply connected open set.
 - (c) State the Cauchy-Goursat theorem.
- (2) State and prove the Cauchy integral formulas.
- (3) State and prove the fundamental theorem of Calculus.
- (4) Define $f(z) = f(x + iy) = x^2 + iy^2$.
 - (a) Where is f continuous?
 - (b) Where is f differentiable?
 - (c) Where is f analytic?
- (5) Compute all possible values of the following. Give your answers in the form $a + bi$.
 - (a) 3^{i+1} .
 - (b) $\log(\sin(i + 1))$.
- (6) Let Γ_R be the upper half of the circle $|z - 2| = R$ oriented in the counter-clockwise direction.

Show that

$$\lim_{R \rightarrow \infty} \int_{\Gamma_R} \frac{z^2 - 3z - 6}{z^4 + 9} dz = 0.$$

- (7) Let C_R be the circle $|z - 1| = R$. Compute

$$\int_{C_R} \frac{\text{Log}(z - 1)}{z - 1} dz.$$

Is it independent of R ? Why?

- (8) Let C be the circle of radius 3 with center $3i$.

(a) Compute

$$\int_C \frac{e^z}{z^2 + 9} dz.$$

- (b) Give an example of another closed contour C' along which the integral of the function above has a different value.