MATH 120A

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March 10 2017: Practice problems for the final exam

Remember that you can use both Cartesian and polar coordinates.

Please explain or prove all your assertions and show your work. Please state all the definitions, propositions, theorems, lemmas that you use precisely.

Please make sure to review all definitions, statements of theorems, proofs done in class, practice problems for the midterms and homework problems.

There will be ten questions on the final: 1 about definitions, 1 about statements of theorems, 2 proofs, and 6 problems.

Good luck!

(1) Let C be the square whose corners are at the four points $z = \pm 3 \pm 3i$. Let I be the integral

$$\int_C \frac{e^z}{z+2i} dz,$$

where C is oriented counterclockwise.

(a) Determine an upper bound for the modulus $|I|$ without evaluating the integral.

(b) Compute the exact value of I.

(2) Suppose the function $F(z)$ is analytic on and inside the circle of radius R and center z_0. Let N be an integer and consider the integral

$$\int_0^{2\pi} e^{iN\theta} F(z_0 + Re^{i\theta}) d\theta.$$

(a) Re-express this integral as a contour integral of an appropriate function of z around the contour C.

(b) Evaluate the integral for all integer values of N (positive, negative, and zero).

(3) (a) Compute the Taylor series expansion of the function $Log(2z+1)$ around the point $z_0 = i$.

(b) What is the radius of the largest circle centered at z_0 in which the series converges to $Log(2z+1)$?

(4) The function $z^{i-\sqrt{2}}$ is multivalued; let $f(z)$ be the principal branch of this function, defined by using the principal argument of z. Compute

$$\int_C f(z) dz,$$

where the contour C is the circle of radius R centered at the origin, traced counterclockwise.

(5) Evaluate the contour integral

$$\int_C \frac{\cos z}{(z-\pi)^2(z-i)} dz,$$

where

(a) C is the circle of radius 1 around $z_0 = \pi$,

(b) C is the circle of radius 2 around the origin,

(c) C is the circle of radius 4 around the origin.

All circles are oriented counterclockwise.

(6) Let $f(z) = u(x,y) + iv(x,y)$ be an entire function. Here, as usual, $z = x + iy$, and the functions $u(x,y)$ and $v(x,y)$ are real-valued. Suppose that there are nonzero real constants a, b, c such that $au(x,y) + bv(x,y) = c$ for all (x,y). Show that $f(z)$ must be constant.