MATH 150A
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Please prove all your assertions and state all the definitions, propositions, theorems, lemmas that you use precisely. Good luck!
(1) Compute the Frenet apparatus (meaning Frenet frames, curvature and torsion) and the arclength function of the curve

$$
\alpha(t)=\left(3 t-t^{3}, 3 t^{2}, 3 t+t^{3}\right)
$$

Write the Frenet matrix giving the derivatives of the vectors of the Frenet frame. Write the equations of the evolute and the involute (with origin $t=0$ ).
(2) Prove that a curve $\alpha(t)$ is a line if and only if $\alpha^{\prime}$ and $\alpha^{\prime \prime}$ are always linearly dependent.
(3) Prove that a curve is contained in a plane if and only if its torsion is always 0 .
(4) Prove that the curvature of a plane curve $\alpha(t)=(x(t), y(t))$ is given by the formula

$$
\frac{\left|x^{\prime} y^{\prime \prime}-x^{\prime \prime} y^{\prime}\right|}{\left(\sqrt{x^{\prime 2}+y^{\prime 2}}\right)^{3}} .
$$

(5) The helicoid: Consider the map

$$
\varphi(u, v)=(u \cos v, u \sin v, b v)
$$

for $b \neq 0$.
(a) Prove that this is a coordinate patch (i.e., it is one-to-one and the vectors $\varphi_{u}$ and $\varphi_{v}$ are always linearly independent). What is its domain?
(b) Write equations for the tangent planes to the image surface $M$.
(c) Describe its parameter curves $u=$ constant and $v=$ constant.
(d) Write an implicit equation $g(x, y, z)=c$ for the surface it describes.
(e) Compute its shape operator and write it as a $2 \times 2$ matrix, using the basis $\left\{\varphi_{u}, \varphi_{v}\right\}$ of the tangent space of $M$ (at any given point).

