(1) A ruled surface \( \varphi(u,v) = \beta(u) + v\delta(u) \) is developable if its unit normal \( U \) is constant along the rulings, i.e., \( U_v = 0 \). Show that a ruled surface is developable if and only if its Gauss curvature is 0.

(2) Without using the previous exercise show that cones and cylinders are developable.

(3) Let \( M \) be a surface and \( p \) and \( q \) two points on \( M \). Let \( \alpha \) be a piecewise regular curve on \( M \) from \( p \) to \( q \). Prove that if \( \alpha \) has the shortest length of all curves from \( p \) to \( q \) on \( M \), then \( \alpha \) is a geodesic.

Hint: One can define a distance on \( M \) by letting the distance between two points be the length of the shortest curve joining them. To show that \( \alpha \) has no corners, use the following result: given \( p \in M \), there exists \( \epsilon = \epsilon_p > 0 \) such that every point \( q \) of distance \( \epsilon \) from \( p \) has a normal neighborhood of radius \( \epsilon \) (you can find a proof in DoCarmo’s book).

(4) For a non-unit speed curve \( \alpha(t) \) with speed \( \nu(t) \), prove that

\[
\alpha'' = \nu'T + \kappa_g\nu^2U \times T + (\alpha'' \cdot U)U
\]

where \( \kappa_g \) is the geodesic curvature of \( \alpha \).