MATH 150B

Elham Izadi

May 18 2017: Practice problems for second midterm

Please explain or prove all your assertions and show your work. Please state all the definitions, propositions, theorems, lemmas that you use precisely.

Please make sure to review all definitions, statements of theorems, proofs done in class, practice problems and homework problems. The exercises below are complementary to the rest of the material and do not replace the lectures, homework problems, etc.

There will be five questions on the midterm: 1 about definitions and statements of theorems, 1 proof, and 3 problems.

Although homework 6 is due after the test, its material is covered by the test: please make sure to do homework 6 before the midterm.

Good luck!

(1) Compute the dual 1-forms, connection form $\omega_{12}$, and Gaussian curvature for the associated frame fields of the following orthogonal coordinate charts.

  (a) Helicoid: $\varphi(u,v) = (u\cos v, u\sin v, bv)$.

  (b) Paraboloid of revolution: $\varphi(u,v) = (u\cos v, u\sin v, u^2/2)$.

  (c) Cone: $\varphi(u,v) = (u\cos v, u\sin v, au)$.

(2) Let $\varphi : D \to M$ be a coordinate chart such that $E = G$ and $F = 0$.

  (a) Prove that the metric on the image of $\varphi$ is conformal with scale factor $\lambda = \frac{1}{\sqrt{E}} = \frac{1}{\sqrt{G}}$.

  (b) Compute the Gaussian curvature of this metric in terms of $\lambda$.

  (c) Prove that the mean curvature $H$ is zero (i.e., $M$ is minimal) if and only if $\varphi_{uu} + \varphi_{vv} = 0$.

(3) Suppose $\varphi : D \to M$ is a principal chart (i.e., the $u$-parameter and $v$-parameter curves are principal curves). Prove the following

  (a)

$$\omega_{13} = \frac{l}{\sqrt{E}}du, \quad \omega_{23} = \frac{n}{\sqrt{G}}dv.$$

  (b)

$$l_v = HE_v, \quad n_u = HG_u.$$

(4) Let $P$ be the Poincaré half plane and let $\alpha$ be the curve $\alpha(t) = (r\cos t, r\sin t), 0 < t < \pi, r$ constant.

(a) Show that the speed of $\alpha$ is $1/\sin t$.

(b) Deduce that the Poincaré length of $\alpha$ is infinite.

(c) Fine the area of the region between $\alpha$ and the $x$-axis.