We saw that $R$ is Zariski closed in $\text{MP}$. Let $R'$ be the parameter space of triples $(f, C', D)$ where $f: C' \to D$ is a map of degree $\geq 2$ to a curve $D$ of genus $\geq 2$ from $C'$ of genus $h$.

We have the natural map $R' \to R \subset \text{MP}$

$$(f, C, D) \mapsto C'$$

$C'$ induces a map $\eta: U \to \text{MP}$

$$t \mapsto [C_t]$$

$$U_B \subset U \quad \eta(U_B) \subset R$$

$$\{ t \in U \mid A_t \text{ isog. to } B \times E_t \}$$

$U_B$ is dense in $U \implies \eta(U) \subset R$
Theorem of de Franchis and Sevèri: \( K : \mathbb{R}^l \to \mathbb{R} \) is finite to one. \( K^{-1}(\eta(U)) \to \eta(U) \) finite to 1.

Shrink \( U \) so that \( K^{-1}(\eta(U)) = \bigsqcup_{i=1}^{\cdots N} V_i \), with \( V_i \to \eta(U) \) a homeomorphism.

On \( U_B \) we have the map:

\[
L : U_B \to \bigsqcup_{i=1}^{\cdots N} V_i = K^{-1}(\eta(U))
\]

\[
(L(U_B))_i : = L^{-1}(V_i)
\]

\( U_B \) dense in \( U \) \( \Rightarrow \) \( \eta(U_B) \) is dense in \( \eta(U) \)

\( \Rightarrow \exists i_0 \) s.t. \( L(U_B)_{i_0} \) is dense in \( V_{i_0} \).

Now lift \( U \to \eta(U) \to V_{i_0} \subset \mathbb{R}^l \).
Shrink $U$ so we have universal families

$G'_U \rightarrow D_U \Rightarrow \text{Pic} \circ D_U \rightarrow \text{Pic} \circ G'_U$

and $C'_U \rightarrow A_U \Rightarrow \text{Pic} \circ A_U \rightarrow \text{Pic} \circ C'_U$

$A_t = \text{Pic} \circ A_t \rightarrow J_{C'_t}$

$J_{D_t} \Rightarrow \downarrow U$

Shrink $U$ so that it is simply connected. Then the bundles of $H_1(A_t, \mathbb{Z})$, $H_1(C'_t, \mathbb{Z})$, $H_1(D_t, \mathbb{Z})$ are topologically trivial.
The families of lattices are isomorphic to
\[ H_1(A_{t_0}, \mathbb{Z}) \times U \rightarrow H_1(C'_{t_0}, \mathbb{Z}) \times U \rightarrow H_1(D_{t_0}, \mathbb{Z}) \times U \]

So the maps on lattices are constant \( \Rightarrow \)
\[ \dim (\text{Im } A_t \cap \text{Im } J D_{t}) \text{ is constant} \] in \( J C'_{t} \).

Since \( A_t \) is generically simple,
\[ \text{Im } A_t \cap \text{Im } J D_{t} = \begin{cases} \{ 0 \} & \text{if } a = \text{Im } A_t \\ a \in A_t \end{cases} \]
\[ A_t \rightarrow J C'_{t} \quad \text{on } \Delta < U_B : \quad \text{Im } J D_{t} \supset \text{Im } B \]

\[ J D_{t} \rightarrow \text{Im } A_t \cap \text{Im } J D_{t} \neq \{ 0 \} \]
\[ \Rightarrow \text{Im } A_t \subset \text{Im } J D_{t} \]
On $\Delta \subset \mathbb{B}$: $J \theta$ is constant but $A_{\theta} \Rightarrow E_{\theta}$ variable contradiction. \( \square \)

**The rigidity theorem:** A abelian variety of dim $g \geq 2$.

$K = K(A) = A/\pm 1$ the Kummer variety of $A$.

$\pi: A \to K$ the quotient map.

A smooth curve of genus $g$, $\varphi: C \to K$ a morphism s.t. $\pi^{-1}(\varphi(C)) \subset A$ generates $A$ as a group.

If $g < q-1$, then $\varphi(C)$ is rigid in $K$.

This means that any deformation of $\varphi(C)$ in $K$ is contained in $\varphi(C)$. In other words: $\forall$ any the image of $C$ in $K$ is a curve.