MATH 20B, Fall 2014
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Friday November 14: Practice problems for Second midterm

Please show your work for all the problems. Answers alone will receive no credit.
Please write your solutions clearly and legibly; no credit will be given for illegible solutions.
Some of the problems below are from old midterms from previous years, some are from the book.
For more practice, also do more problems from the relevant sections of the book.

(1) Compute the following integrals (if you use reduction formulas, show how you derive them
using integration by parts)
(a) Do this one in two different ways, using complex numbers and using integration by parts.
\[ \int e^{5x} \cos(3x) \, dx \]
(b) \[ \int \sin^3(x) \cos^4(x) \, dx \]
(c) \[ \int \cot^4(x) \csc(x) \, dx \]
(d) \[ \int \cot^4(x) \csc^2(x) \, dx \]
(e) Do this one in two different ways, with the double angle formulas and integration by
parts.
\[ \int \sin^4(x) \cos^4(x) \, dx \]
(f) \[ \int \frac{dx}{\sin(x)} \]
(g) \[ \int \frac{(4x^4 - 20x^3 + 30x^2 - 110x + 115) \, dx}{(x - 3)^2(2x^2 + 5)} \]
(h) \[ \int \frac{(10x^2 - 28x - 156) \, dx}{(x + 3)(x + 5)(2x - 3)} \]
(i) \[ \int \frac{(x^2 + 3) \, dx}{(x^2 + 2x + 3)^2} \]
(2) Determine whether the following improper integrals converge or diverge
(a) \[ \int_{-\infty}^{0} \frac{e^x}{\sqrt{-x}} \, dx \]
(b) \[ \int_{0}^{\infty} \frac{e^x}{\sqrt{x}} \, dx \]
(c) \[ \int_{-\infty}^{\infty} \frac{dx}{(x^2 - 1)^{\frac{3}{2}}} \]

(3) (a) Write the general trapezoidal and midpoint sums approximating the integral of the
function \( \sqrt{x^4 + 1} \) over the interval \([1, 2]\).
(b) Write the sums for \( n = 5 \).

(4) Find all the (complex) sixth roots of 27 and plot them in the complex plane.

(5) Derive a trigonometric identity between sine and cosine of 4\( \theta \) and sine and cosine of \( \theta \) using
complex numbers.

(6) (a) Put \( t = \tan(\frac{\theta}{2}) \) and show that
\[
\sin(\theta) = \frac{2t}{1 + t^2}, \quad \cos(\theta) = \frac{1 - t^2}{1 + t^2}.
\]
(b) Use the above to convert all the trigonometric integrals of problem 1 into integrals of
rational functions. Can you compute the integrals using the method of partial fractions?

(7) (a) Plot the curves of equation \( r = \sin(2\theta) \) and \( r = 4\cos(\theta) \).
(b) Compute the area between the curves of equation \( r = \sin(2\theta) \) and \( r = 4\cos(\theta) \).