This is only a summary. Please make sure to read the book, review your class notes, the homework problems and the practice problems.

Chapter 5:

1. (§5.2) Riemann sums, the definition of \( \int_{a}^{b} f(x) \, dx \)
2. (§5.2) Basic properties of definite integrals
   \[
   \int_{a}^{b} [f(x) + g(x)] \, dx = \int_{a}^{b} f(x) \, dx + \int_{a}^{b} g(x) \, dx \\
   \int_{a}^{b} f(x) \, dx = -\int_{b}^{a} f(x) \, dx \\
   \int_{a}^{b} f(x) \, dx = \int_{c}^{a} f(x) \, dx + \int_{c}^{b} f(x) \, dx.
   \]
3. (§5.2) Comparison theorem: if \( f(x) \leq g(x) \) for \( a \leq x \leq b \) then \( \int_{a}^{b} f(x) \, dx \leq \int_{a}^{b} g(x) \, dx \).
4. (§5.3) The Fundamental Theorem of Calculus, Part I: if \( f \) is continuous on \([a, b]\) and \( F'(x) = f(x) \), then \( \int_{a}^{b} f(x) \, dx = F(b) - F(a) \).
5. (§5.4) The Fundamental Theorem of Calculus, Part II: if \( f \) is continuous on \([a, b]\), and \( F(x) = \int_{a}^{x} f(t) \, dt \), then \( F \) is differentiable on \([a, b]\) and \( F'(x) = f(x) \). Combining this with the chain rule: if \( u \) is a differentiable function, then
   \[
   \frac{d}{dx} \int_{a}^{u(x)} f(t) \, dt = f(u(x))u'(x).
   \]
6. (§5.5) Derivatives describe rates of change; the fundamental theorem of calculus tells us that the total change in a quantity is the integral of its rate of change.
7. (§5.6) Substitution (the chain rule in reverse):
   \[
   \int f(u(x))u'(x) \, dx = \int f(u) \, du, \quad \int_{a}^{b} f(u(x))u'(x) \, dx = \int_{u(a)}^{u(b)} f(u) \, du.
   \]

Chapter 6:

1. (§6.1) The area between the curves \( y = f(x) \) and \( y = g(x) \) over the interval \([a, b]\) is \( \int_{a}^{b} [f(x) - g(x)] \, dx \) (assuming \( f \geq g \) on \([a, b]\)).
2. (§6.2) The washer method. If \( \mathcal{V} \) is a 3-dimensional region whose slices perpendicular to some axis (labeled \( x \)) have area \( A(x) \), and if the whole region \( \mathcal{V} \) extends from \( x = a \) to \( x = b \), then \( \text{Vol}(\mathcal{V}) = \int_{a}^{b} A(x) \, dx \).
3. (§6.3) If the region \( \mathcal{V} \) is a volume of revolution of a curve \( y = f(x) \) around the x-axis over \( a \leq x \leq b \), then the slices have area \( A(x) = \pi f(x)^2 \), and so the volume of the region is
   \[
   \text{Volume} = \pi \int_{a}^{b} f(x)^2 \, dx.
   \]
If the region is between two curves $y = f(x)$ and $y = g(x)$, where $f(x) \geq g(x)$, then the slices are annuli with area $A(x) = \pi f(x)^2 - \pi g(x)^2$, so the volume of the region is

$$\text{Volume} = \pi \int_a^b [f(x)^2 - g(x)^2] \, dx.$$ 

If we revolve around the $y$-axis instead, we must first re-express the curve $y = f(x)$ as a function $x = g(y)$, and then use the same formulas interchanging $x$ and $y$. More generally, if we use an axis like the line $x = -2$, the radii need to be adjusted by adding 2. In general, just use the loaf of bread method to figure out what integral to calculate.

(4) (§6.2) The mean or average value of a function $f$ on $[a, b]$ is

$$\frac{1}{b-a} \int_a^b f(x) \, dx.$$

The Mean Value Theorem for integrals says that, if $f$ is continuous on $[a, b]$, then there is some point $c \in [a, b]$ where $f(c)$ is equal to the average value of $f$ on $[a, b]$.

Chapter 7:

(1) (§7.1) Integration by Parts (product rule in reverse):

$$\int u(x)v'(x) \, dx = u(x)v(x) - \int u'(x)v(x) \, dx,$$

$$\int_a^b u(x)v'(x) \, dx = u(b)v(b) - u(a)v(a) - \int_a^b u'(x)v(x) \, dx.$$ 

When the integrand is a product $f(x)g(x)$, best to set $u(x)$ to be whichever function gets simpler when differentiating (say $f(x)$), provided the other function is something you can integrate (since you need to find $v(x)$ by setting $v'(x) = g(x)$). A sometimes useful trick is to apply this when $g(x) = 1$; this allows us to integrate functions like $\ln x$.

Chapter 11:

(1) (§11.3) Polar coordinates $(r, \theta)$ in the plane. Convert back and forth with Cartesian coordinates by

$$x = r \cos \theta \quad r = \sqrt{x^2 + y^2}$$

$$y = r \sin \theta \quad \tan \theta = \frac{y}{x} (x \neq 0)$$

Caution: the tan function is not one-to-one, so you cannot simply take $\tan^{-1}$. Always draw the picture to inform which branch of the tan function your angle lies on (it may be off by $\pi$, when $x < 0$).

(2) (§11.4) Area in polar coordinates: the area “under” the polar curve $r = f(\theta)$, with $\theta \in [\alpha, \beta]$ (i.e. the polar sector $\{(r, \theta) : \alpha \leq \theta \leq \beta, 0 \leq r \leq f(\theta)\}$) is given by

$$\frac{1}{2} \int_{\alpha}^{\beta} f(\theta)^2 \, d\theta.$$ 

If $f \geq g \geq 0$ then the area between $r = f(\theta)$ and $r = g(\theta)$ is given by

$$\frac{1}{2} \int_{\alpha}^{\beta} [f(\theta)^2 - g(\theta)^2] \, d\theta.$$