Please make sure to read the book, review your class notes, the homework problems and the practice problems.

The midterm will cover Sections 5.2 - 5.6, 6.1 - 6.3, 7.1, 7.2, 7.4 - 7.6, 7.8, 11.3, 11.4 from the book and Supplement 1, 2, 3, 4, 5, with an emphasis on Sections 7.2, 7.4 - 7.6, 7.8, and Supplement 1, 2, 3, 4, 5.

You might also find the following useful when preparing for your exam.

Chapter 7:

(1) (§7.2) Trig integrals with powers of sin and cos at least one of which is odd. For example

\[
\int \sin^3 x \cos^4 x \, dx = \int \cos^4 x \sin^2 x \cdot \sin x \, dx
\]

Substitute \( u = \cos x \), so that \( du = -\sin x \, dx \). Use the Pythagorean theorem \( \sin^2 x = 1 - \cos^2 x \), to get

\[
\int u^4 (1 - u^2) (-du) = \int (u^6 - u^4) \, du = \frac{1}{7} u^7 - \frac{1}{5} u^5 + C = \frac{1}{7} \cos^7 x - \frac{1}{5} \cos^5 x + C.
\]

(2) (§7.2) Trig integrals with even powers of sin and cos: first use the Pythagorean theorem to express in terms of just sin or just cos. Then use integration by part to get a recursion. For example

\[
\int \sin^4 x \cos^2 x \, dx = \int \sin^4 x (1 - \sin^2 x) \, dx = \int \sin^4 x \, dx - \int \sin^6 x \, dx.
\]

Proceeding with the first integral,

\[
I = \int \sin^4 x \, dx = \int \sin^3 x \cdot \sin x \, dx = \int U \, dV
\]

where \( U = \sin^3 x \) so \( dU = 3 \sin^2 x \cos x \, dx \), and \( dV = \sin x \, dx \) so \( V = -\cos x \). Thus

\[
I = UV - \int V \, dU = -\sin^3 x \cos x - \int (-\cos x)(3 \sin^2 x \cos x) \, dx
\]

\[
= -\sin^3 x \cos x + 3 \int \sin^2 x \cos^2 x \, dx.
\]

Using Pythagoras again,

\[
\int \sin^2 x \cos^2 x \, dx = \int \sin^2 x (1 - \sin^2 x) \, dx = \int \sin^2 x \, dx - \int \sin^4 x \, dx
\]

\[
= \int \sin^2 x \, dx - I.
\]

This yields

\[
I = -\sin^4 x \cos x + 3 \left( \int \sin^2 x \, dx - I \right).
\]

Solving this for \( I \) we get

\[
I = -\frac{1}{4} \sin^4 x \cos x + \frac{3}{4} \int \sin^2 x \, dx.
\]
We now proceed with the same integration by parts reduction to evaluate \( \int \sin^2 x \, dx \). When all done, we use the same approach (repeatedly) to evaluate \( \int \sin^6 x \, dx \), to get the full final answer
\[
\int \sin^4 x \cos^2 x \, dx = \frac{1}{6} \sin^5 x \cos x - \frac{1}{24} \sin^4 x \cos x - \frac{1}{6} \cos x \sin x + \frac{1}{16} x + C.
\]

You can apply a similar method for computing integrals of powers of \( \sec \), powers of \( \tan \) and their products. Compute \( \int \sec^5(x) \, dx \) using integration by parts to reduce the power of \( \sec \) as above.

Chapter 7:
1. (§7.1) Integration by Parts (product rule in reverse):
\[
\int u(x)v'(x) \, dx = u(x)v(x) - \int u'(x)v(x) \, dx,
\]
\[
\int_a^b u(x)v'(x) \, dx = u(b)v(b) - u(a)v(a) - \int_a^b u'(x)v(x) \, dx.
\]
When the integrand is a product \( f(x)g(x) \), best to set \( u(x) \) to be whichever function gets simpler when differentiating (say \( f(x) \)), provided the other function is something you can integrate (since you need to find \( v(x) \) by setting \( v'(x) = g(x) \)). A sometimes useful trick is to apply this when \( g(x) = 1 \); this allows us to integrate functions like \( \ln x \).

Chapter 11:
1. (§11.3) Polar coordinates \((r, \theta)\) in the plane. Convert back and forth with Cartesian coordinates by
\[
x = r \cos \theta \quad r = \sqrt{x^2 + y^2} \\
y = r \sin \theta \quad \tan \theta = \frac{y}{x} (x \neq 0)
\]
Caution: the tan function is not one-to-one, so you cannot simply take \( \tan^{-1} \). Always draw the picture to inform which branch of the tan function your angle lies on (it may be off by \( \pi \), when \( x < 0 \)).

2. (§11.4) Area in polar coordinates: the area “under” the polar curve \( r = f(\theta) \), with \( \theta \in [\alpha, \beta] \) (i.e. the polar sector \( \{(r, \theta): \alpha \leq \theta \leq \beta, 0 \leq r \leq f(\theta)\}\) is given by
\[
\frac{1}{2} \int_\alpha^\beta f(\theta)^2 \, d\theta.
\]
If \( f \geq g \geq 0 \) then the area between \( r = f(\theta) \) and \( r = g(\theta) \) is given by
\[
\frac{1}{2} \int_\alpha^\beta [f(\theta)^2 - g(\theta)^2] \, d\theta.
\]