Not all solutions are complete but hopefully, there is enough here that you can complete the solution on your own. Please also see the list of topics for the second midterm for some useful calculations.

(1)(a) Done in class.
(1)(b) Make the substitution \( u = \cos(x) \)
(1)(c) This was a homework problem which we also did in class and in office hours today. Write \( \cot^2(x) = \csc^2(x) - 1 \) and substitute to obtain

\[
\int \csc^5(x)dx - 2 \int \csc^3(x)dx + \int \csc(x)dx
\]

Then reduce the power of cosecant by integration by parts. For instance, put \( f' = -\csc^2(x) \) and \( g = \csc^3(x) \). Then \( f = \cot(x) \) and \( g' = -3\csc^3(x)\cot(x) \). This gives

\[
\int \csc^5(x)dx = \csc^3(x)\cot(x) - 3 \int \csc^3(x)\cot^2(x)dx
\]

Again use the formula \( \cot^2(x) = \csc^2(x) - 1 \) to get

\[
\int \csc^5(x)dx = \csc^3(x)\cot(x) - 3 \int \csc^5(x)dx + \int \csc^3(x)dx
\]

Now this is an equation in \( \int \csc^5(x)dx \), which we solve to get

\[
\int \csc^5(x)dx = \frac{1}{4} \left( \csc^3(x)\cot(x) + \int \csc^3(x)dx \right).
\]

Repeat to compute the \( \int \csc^3(x)dx \) in terms of \( \int \csc(x)dx = \int \frac{dx}{\sin(x)} \).

(1)(d) Make the substitution \( u = \cot(x) \).

(1)(g) The rational function is proper so we do not need to do Euclidean division. The partial fractions expansion is

\[
\frac{4x^4 - 20x^3 + 30x^2 - 110x + 115}{(x - 3)(2x^2 + 5)} = \frac{2}{x - 3} - \frac{7}{(x - 3)^2} + \frac{10}{2x^2 + 5}
\]

To integrate the quadratic term, make the substitution \( \sqrt{2} x = \sqrt{5} \tan(t) \).

(1)(h) As above, the partial fractions expansion is

\[
\frac{10x^2 - 28x - 156}{(x + 3)(x + 5)(2x - 3)} = \frac{1}{x + 3} + \frac{9}{x + 5} - \frac{6}{2x - 3}
\]

(1)(i)

\[
\frac{x^2 + 3}{(x^2 + 2x + 3)^2} = \frac{x^2 + 2x + 3 - 2x}{(x^2 + 2x + 3)^2} = \frac{1}{x^2 + 2x + 3} - \frac{2x}{(x^2 + 2x + 3)^2}
\]
So

\[
\int \frac{(x^2 + 3)dx}{(x^2 + 2x + 3)^2} = \int \frac{dx}{x^2 + 2x + 3} - \int \frac{2xdx}{(x^2 + 2x + 3)^2}
\]

Complete the square to get \(x^2 + 2x + 3 = (x + 1)^2 + 2\) and make the substitution \(u = x + 1\) (\(du = dx\)) to get

\[
\int \frac{(x^2 + 3)dx}{(x^2 + 2x + 3)^2} = \int \frac{du}{u^2 + 2} - \int \frac{2(u - 1)du}{(u^2 + 2)^2} = \int \frac{du}{u^2 + 2} - \int \frac{2udu}{(u^2 + 2)^2} + \int \frac{2du}{(u^2 + 2)^2}
\]

For the middle integral make the substitution \(v = u^2 + 2\). For the first and third integrals make the substitution \(u = \sqrt{2}\tan(t)\).