$$(da)_p(\alpha'(0)) = \left(\frac{\partial y_j}{\partial x_i}(0)\right)_{\substack{1\le j\le n\\ 1\le i\le n}} \begin{pmatrix} x_1'(0) \\ \vdots \\ x_n'(0) \end{pmatrix}$$

First application: $V_1 \xrightarrow[\varphi_1]{} M \qquad V_2 \xrightarrow[\varphi_2]{} M$

two coordinate charts.

choose $M_1 = \varphi_1^{-1}(\varphi_1(V_1) \cap \varphi_2(V_2)) \subset V_1 \subset \mathbb{R}^n \quad x_1,\dots,x_n$

$M_2 = \varphi_2^{-1}(\varphi_1(V_1) \cap \varphi_2(V_2)) \subset V_2 \subset \mathbb{R}^n \quad y_1,\dots,y_n$

$a := \varphi_2^{-1}\varphi_1 : M_1 \longrightarrow M_2$

$(da)_p$ at $p \in V_1$ has matrix $\left(\frac{\partial y_j}{\partial x_i}\right)_{\substack{1\le j\le n\\ 1\le i\le n}}$

__Definition:__ $a: M_1 \longrightarrow M_2$ is a diffeomorphism if a is a differentiable bijection and a^{-1} is also differentiable. a is a local diffeomorphism if $\forall\, p \in M_1$, $\exists$ an open set U of M_1 containing p s.t. $a|_U : U \longrightarrow a(U)$ is a diffeomorphism and $a(U)$ is open in M_2.

__Theorem__ (almost 2.10): If $\varphi : M_1 \longrightarrow M_2$ is differentiable, then φ is a local diffeomorphism iff $\forall\, p \in M_1$ $(d\varphi)_p$ is an isomorphism (of vector spaces) $T_p M_1 \to T_{\varphi(p)} M_2$.

φ is a diffeomorphism iff φ is a local diffeomorphism and φ is bijective.